

1.1 Relations and Functions

When mathematicians, economists, pollsters, and scientists recognize a relationship between items in the world around them, they try to model the relationship with an equation. Does the value of one measured quantity guarantee a unique value for the second related quantity? This question defines the difference between a relation and a function.

Relation – an identified pattern between two variables that may be represented as ordered pairs, a table of values, a graph, or an equation

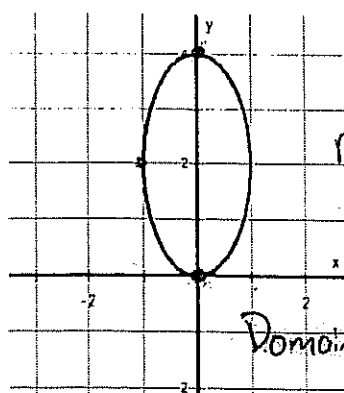
Function – a relation in which each value of the independent variable (the first coordinate) corresponds to *exactly one value* of the dependent variable (the second coordinate)

The vertical line test can be used to determine whether a relation is a function.

Vertical line test

- A method of determining whether a relation is a function
- If every vertical line intersects the relation at only one point, then the relation is a function

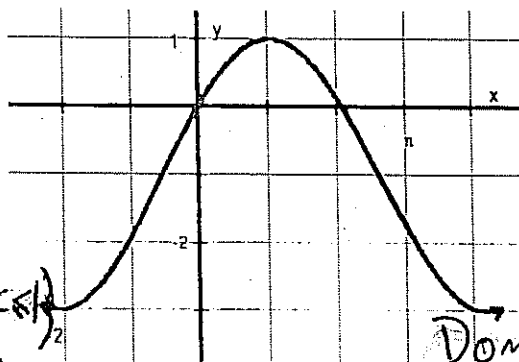
eg. Indicate whether or not each of the following relations are functions.



not a function

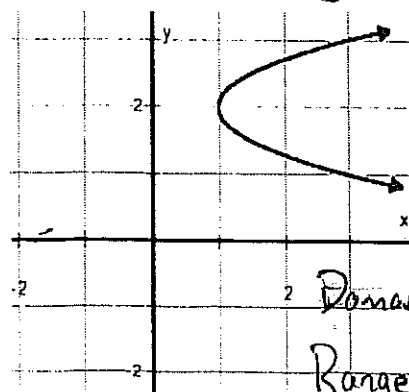
Domain: $\{x \in \mathbb{R} \mid -1 \leq x \leq 1\}$

Range: $\{y \in \mathbb{R} \mid 0 \leq y \leq 4\}$ such that



function

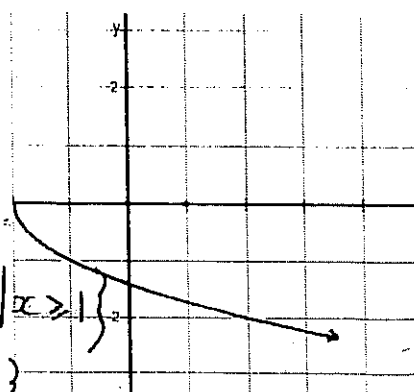
Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R} \mid y \leq 1\}$



not a function

Domain: $\{x \in \mathbb{R} \mid x \geq 1\}$

Range: $\{y \in \mathbb{R}\}$



function

Domain: $\{x \in \mathbb{R} \mid x \geq -2\}$
Range: $\{y \in \mathbb{R} \mid y \leq 0\}$

For any relation, the set of values of the independent variable (often the x-values) is called the domain of the relation.

The set of the corresponding values of the dependent variable (often the y-values) is called the range of the relation.

- For a function, for each given element of the domain there must be exactly one element in the range.

When the equation of a relation is given, the domain and range can be determined by analysing the allowable values from the set of real numbers. \mathbb{R} ← the set of real numbers

Real numbers – a number in the set of all integers, terminating decimals, repeating decimal, non-terminating decimals, and non-repeated decimals, represented by the symbol \mathbb{R} .

Restrictions on the domain occur because division by zero is undefined and because expressions under a radical sign must be greater than or equal to zero. The range can have restrictions too. For example, a quadratic that opens upward will have a minimum value.

*Set notation is used to write the domain and range for a function. For example, for the function $y = x^2 + 2$:

Domain $\{x \in \mathbb{R}\}$ ← \in (in the element of)

Range $\{y \in \mathbb{R}, y \geq 2\}$

← y can be greater than or equal to 2

Homework: p10-12 #1-6, 12, 14