11U Functions Chapter 1, Note 2 – 1.2 – Functions and Function Notation

In this section, you will extend the concept of a function and formalize several notations that are used to represent a function. $y = x + 2$

$f(x) = x + 2$

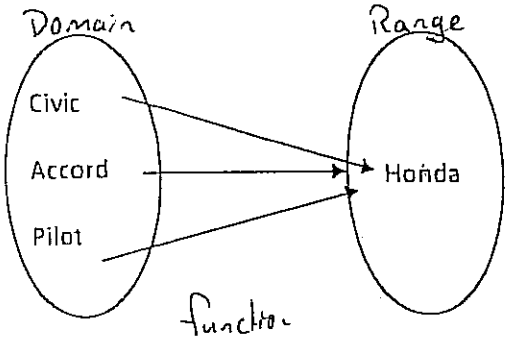
In <u>function notation</u>, the symbol $f(x)$ represents the dependent variable. It indicates that the function f is expressed in terms of the independent variable x.

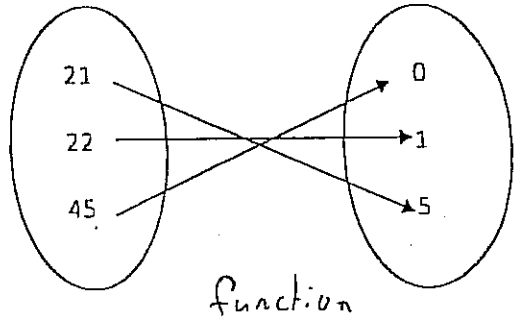
Ex. $y = 3x^2 - 5$ is written as $f(x) = 3x^2 - 5$

The notation $f(3)$ means the value obtained when $x = 3$ is substituted. $f(3)$ is read as "f at 3" or "f of 3."

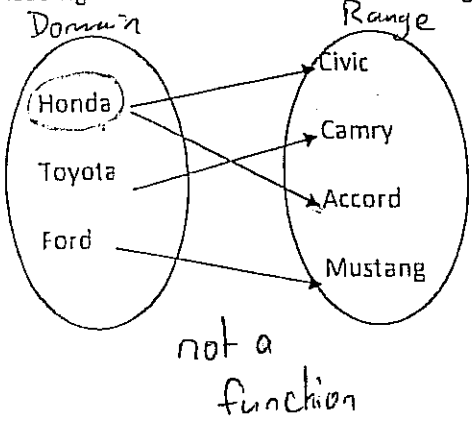
- Letters other than f can be used in function notation.
  Ex. if height is being measured as a function of time, express the function as $h(t)$

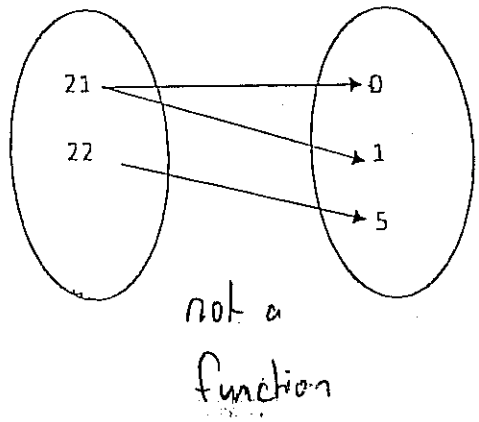
Relations and functions given as ordered pairs can be represented using ***mapping diagrams***. This involves using directed arrows from each value in an oval representing the domain to the corresponding value ~~of~~ of values in an oval representing the range.





In a mapping diagram, a relation is not a function when an element from the domain has two or more arrows leading to different elements of the range.

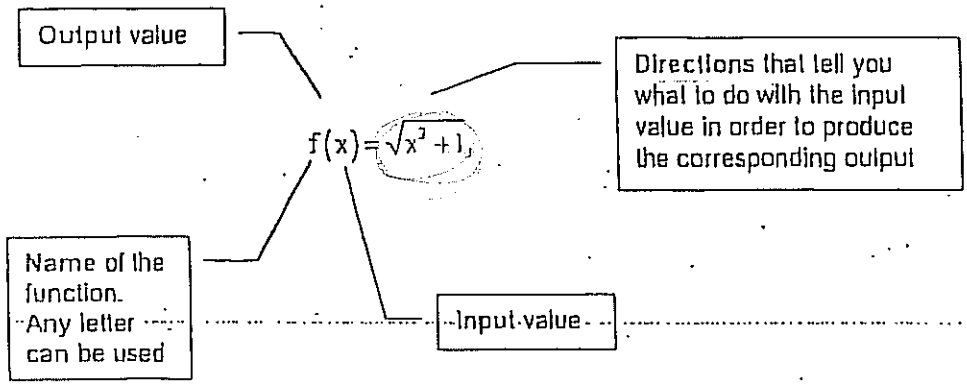




Function Notation

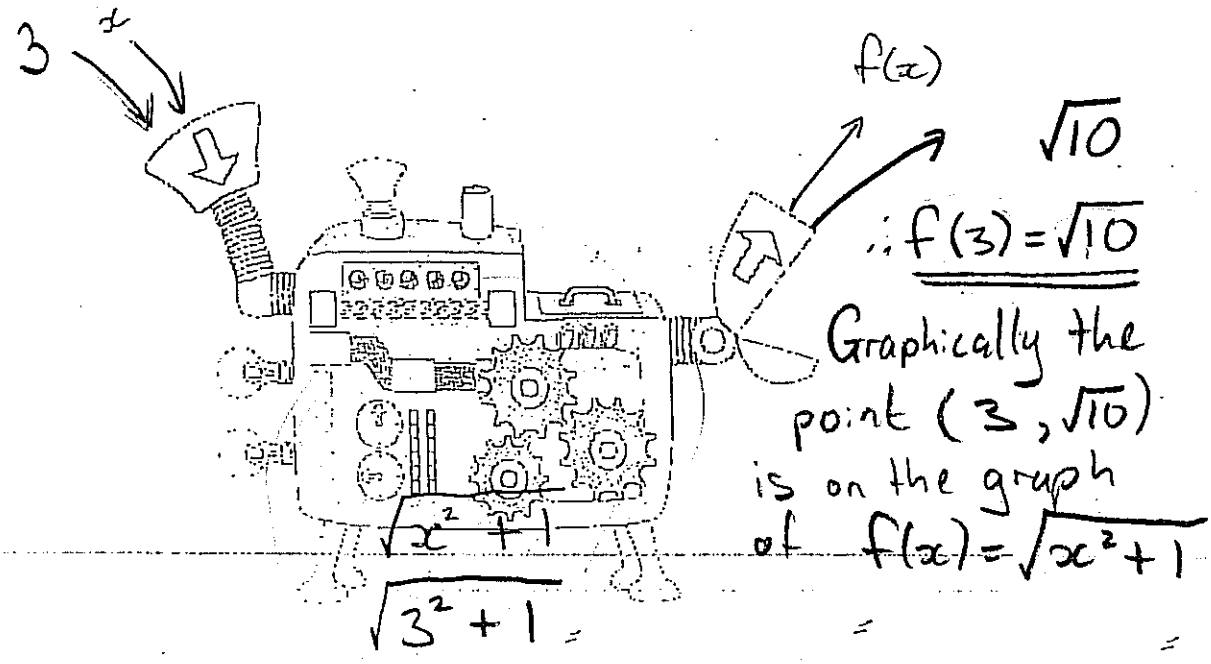
Because functions are used throughout mathematics, function notation, was developed to make it easier to work with them. Function notation can be used even when we do not know the details of a particular relationship.

Suppose a function is given. Let *f* denote a given function and let x represent the input value. *F*(x) denotes the value (output)of the function when the input is x. In other words, *f*(x)=y. Usually a function is represented by a mathematical expression.



* Don't confuse f(x) with multiplication. The entire symbol f(x) represents a function, its input, and its output. It is not the same as an algebraic expression.

The Function Machine:



.x 1) Find f(3) given $f(x)=\sqrt{x^2+1}$

$f(3) = \sqrt{3^2+1}$

$\therefore f(3) = \sqrt{10}$

Ex 2) If $f(x)=3x^2-7x+2$:

a) Find f(-4)

$f(-4) = 3(-4)^2 - 7(-4) + 2$

$= 3(16) - (-28) + 2$

$= 48 + 28 + 2$

$\therefore f(-4) = 78$

b) Find $f(a^2-1)$

$f(a^2-1) = 3(a^2-1)^2 - 7(a^2-1) + 2$

$= 3(a^4-2a^2+1) - 7a^2 + 7 + 2$

$= 3a^4 - 6a^2 + 3 - 7a^2 + 9$

$\therefore f(a^2-1) = 3a^4 - 13a^2 + 12$

$(a^2-1)^2$

$= (a^2-1)(a^2-1)$

$= a^4 - a^2 - a^2 + 1$

$= a^4 - 2a^2 + 1$

Ex 3) Graph $f(x)=x^3$. Then find the following:

a) $f(-1) = (-1)^3$

$= -1$

b) If x = 4, $f(x) = 4^3$

$= 64$

c) If f(x) = 4, x =

$f(x) = x^3$

sub $f(x) = 4 \rightarrow 4 = x^3$

$\sqrt[3]{4} = x$

$\therefore x = \sqrt[3]{4}$

$f(x) = x^3$

$\sqrt[2]{\ } \quad \sqrt{\ }$

$2 \quad 2$

$3 \quad \sqrt[3]{\ }$

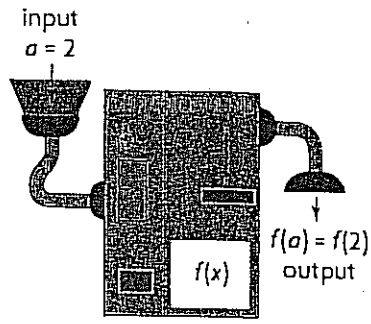
In Summary

Key Idea

- Symbols such as $f(x)$ are called function notation, which is used to represent the value of the dependent variable y for a given value of the independent variable x. For this reason, y and $f(x)$ are interchangeable in the equation of a function, so $y = f(x)$.

Need to Know

- $f(x)$ is read "f at x" or "f of x."
- $f(a)$ represents the value or output of the function when the input is $x = a$. The output depends on the equation of the function. To evaluate $f(a)$, substitute a for x in the equation for $f(x)$.
- $f(a)$ is the y-coordinate of the point on the graph of f with x-coordinate a. For example, if $f(x)$ takes the value 3 at $x = 2$, then $f(2) = 3$ and the point $(2, 3)$ lies on the graph of f.



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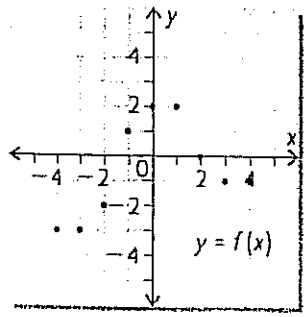
Plus

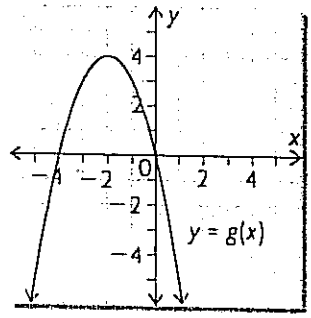
Given $f(x) = x^2$
and $g(x) = f(x+2)$
Sketch the graph
of $g(x)$

CHECK Your Understanding

1. Evaluate, where $f(x) = 2 - 3x$.
   a) $f(2)$
   b) $f(0)$
   c) $f(-4)$
   d) $f\left(\frac{1}{2}\right)$
   e) $f(a)$
   f) $f(3b)$

2. The graphs of $y = f(x)$ and $y = g(x)$ are shown.





Using the graphs, evaluate
   a) $f(1)$
   b) $g(-2)$
   c) $f(4) - g(-2)$
   d) x when $f(x) = -3$

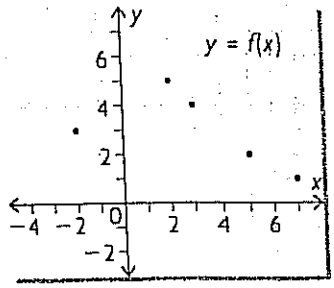
3. Milk is leaking from a carton at a rate of 3 mL/min. There is 1.2 L of milk in the carton at 11:00 a.m.
   a) Use function notation to write an equation for this situation.
   b) How much will be left in the carton at 1:00 p.m.?
   c) At what time will 450 mL of milk be left in the carton?

PRACTISING

4. Evaluate $f(-1)$, $f(3)$, and $f(1.5)$ for
   a) $f(x) = (x - 2)^2 - 1$ b) $f(x) = 2 + 3x - 4x^2$

5. For $f(x) = \frac{1}{2x}$, determine
   a) $f(-3)$ b) $f(0)$ c) $f(1) - f(3)$ d) $f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right)$

6. The graph of $y = f(x)$ is shown at the right.
   a) State the domain and range of f.
   b) Evaluate.
      i) $f(3)$ iii) $f(5 - 3)$
      ii) $f(5)$ iv) $f(5) - f(3)$

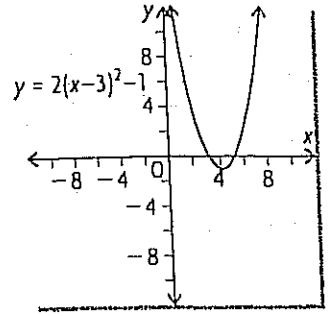


7. For $h(x) = 2x - 5$, determine
   a) $h(a)$ c) $h(3c - 1)$
   b) $h(b + 1)$ d) $h(2 - 5x)$

8. Consider the function $g(t) = 3t + 5$.
   a) Create a table of values and graph the function.
   b) Determine each value.
      i) $g(0)$ iv) $g(2) - g(1)$
      ii) $g(3)$ v) $g(1001) - g(1000)$
      iii) $g(1) - g(0)$ vi) $g(a + 1) - g(a)$

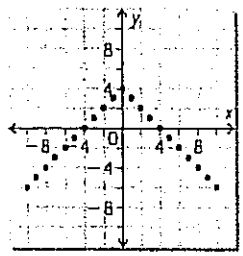
9. Consider the function $f(s) = s^2 - 6s + 9$.
   a) Create a table of values for the function.
   b) Determine each value.
      i) $f(0)$ iv) $f(3)$
      ii) $f(1)$ v) $[f(2) - f(1)] - [f(1) - f(0)]$
      iii) $f(2)$ vi) $[f(3) - f(2)] - [f(2) - f(1)]$
   c) In part (b), what do you notice about the answers to parts (v) and (vi)? Explain why this happens.

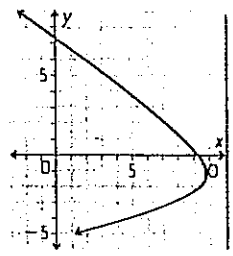
10. The graph at the right shows $f(x) = 2(x - 3)^2 - 1$.
   a) Evaluate $f(-2)$.
   b) What does $f(-2)$ represent on the graph of f?
   c) State the domain and range of the relation.
   d) How do you know that f is a function from its graph?



11. For $g(x) = 4 - 5x$, determine the input for x when the output of $g(x)$ is
   a) -6 b) 2 c) 0 d) $\frac{3}{5}$

8. a) i) 1.25; 2.75 ii) ± 2; 0 iii) 2; -2 iv) 0; $\pm\sqrt{2}$
   b) Functions: (i), (iii)
   c) Graph relation and apply vertical-line test, or solve equation for y and check for multiple values
9. Functions: (a), (b), (d)
10. Not a function; for example, when $x = 6$, $y = 2$ or -2; graph fails vertical-line test
11. Functions: (a), (b)
12. a) domain $= \{x \in R \mid x \geq 0\}$, range $= \{y \in R \mid y \geq 44\}$
    b) Distance cannot be negative, cost cannot be lower than daily rental charge.
    c) Yes, it passes the vertical line test.
13. a) Answers may vary; for example: b) Answers may vary; for example:

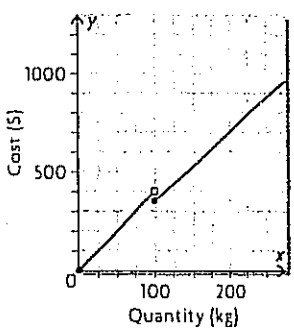




14. Answers may vary; for example:

Definition: A relation with only one y-value for each x-value	Characteristics: A vertical line crosses the graph in at most one place
Function	
Examples: $3x + y = 2$; $y = -2x^2 + 7$	**Non-examples:** $x^2 + y^2 = 16$; $y = \pm\sqrt{x - 7}$

15. a) Each order quantity determines a single cost.
    b) domain $= \{x \in R \mid x \geq 0\}$, range $= \{y \in R \mid y \geq 0\}$
    c)



    d) Answers may vary. For example, the company currently charges less for an order of 100 kg ($350) than for an order of 99 kg ($396). A better system would be for the company to charge $50 plus $3.50 per kilogram for orders of 100 kg or more. This would make the prices strictly increasing as the weight of the order increases.

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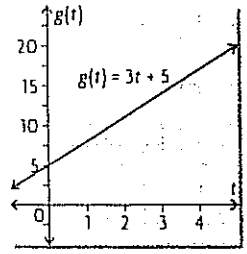
1. a) -4 c) 14 e) $2 - 3a$
   b) 2 d) $\frac{1}{2}$ f) $2 - 9b$
2. a) 2 b) 4 c) -5 d) -3 or -4
3. a) $f(x) = 1200 - 3x$
   b) 840 mL c) 3:10 pm
4. a) 8, 0, -0.75 b) -5, -25, -2.5
5. a) $-\frac{1}{6}$ b) undefined c) $\frac{1}{3}$ d) $2\frac{2}{3}$
6. a) domain $= \{-2, 2, 3, 5, 7\}$, range $= \{1, 2, 3, 4, 5\}$
   b) i) 4 ii) 2 iii) 5 iv) -2
7. a) $2a - 5$ b) $2b - 3$ c) $6c - 7$ d) $-10x - 1$
8. a)

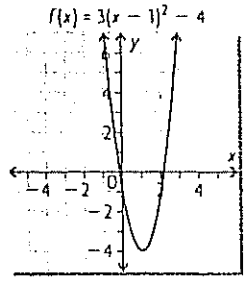
t	$g(t)$
0	5
1	8
2	11
3	14
4	17
5	20



   b) i) 5 ii) 14 iii) 3 iv) 3 v) 3 vi) 3
9. a)

0	9
1	4
2	1
3	0

   b) i) 9 ii) 4 iii) 1 iv) 0 v) 2 vi) 2
   c) They are the same; they represent the second differences, which are constant for a quadratic function.
10. a) 49
    b) The y-coordinate of the point on the graph with x-coordinate -2
    c) domain $= \{x \in R\}$, range $= \{y \in R \mid y \geq -1\}$
    d) It passes the vertical-line test.
11. a) 2 b) 0.4 c) 0.8 d) $\frac{17}{25}$
12. a) $f(x) = 0.15x + 50$ b) $120.80 c) 200 km
13. a) $f(x) = (24 - 3x)x$ b) 45, -195, -60 c) 48
14. $f(x) = 0.0036x(281 - x)$
15. a) $f(x) = 3(x - 1)^2 - 4$



    b) The y-coordinate of the point on the graph with x-coordinate -1; start from -1 on x-axis, move up to curve, then across to y-axis
    c) i) 3 ii) 9 iii) $3x^2 - 4$
16. a) 3, -5 b) 1, -3 c) -1
17. a) $\frac{1}{4}$ b) $\frac{1}{3}$, -1