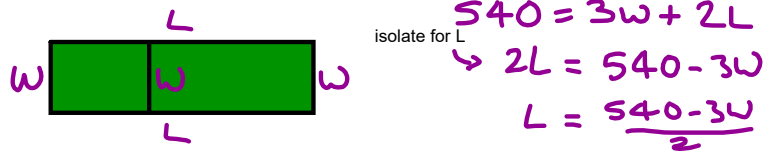


1.4 Domain and Range of a Function

Example 1

- a) A farmer has 540m of fencing to enclose a rectangular area and divide it into two sections as shown:



- a) Write an equation to express the total area enclosed as a function of the width.

$$A = LW$$

$$\text{sub } L = \frac{540 - 3w}{2} \rightarrow = \left(\frac{540 - 3w}{2} \right) w$$

$$\therefore A = \frac{1}{2} w (540 - 3w)$$

- b) Determine the domain and range of this area function. or $A = -\frac{1}{2}w(3w - 540)$

Domain w must be greater than zero for there to be an existing rectangle ($w > 0$)

$3w$ must be less than 540:

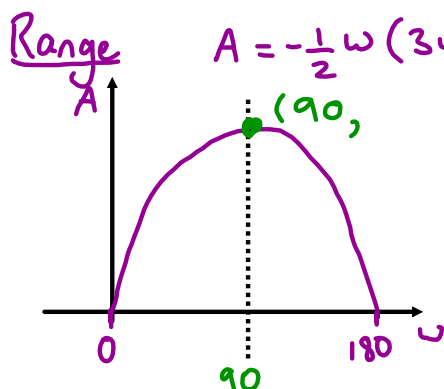
$$3w < 540$$

$$w < \frac{540}{3}$$

$$w < 180$$

$$\therefore \text{Domain} = \{ w \in \mathbb{R} \mid 0 < w < 180 \}$$

↑ lower bound ↑ upper bound



when $w = 90$,

$$A = -\frac{1}{2} (90) (3(90) - 540)$$

$$= -\frac{1}{2} (90) (270 - 540)$$

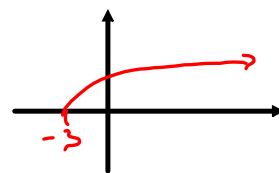
$$= -45 (-270)$$

$$= \underline{12150}$$

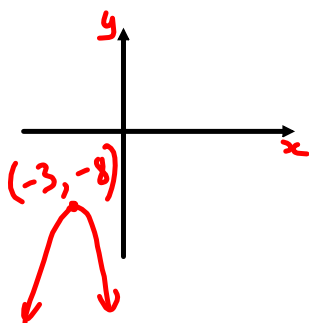
$$\therefore \text{Range} = \{ A \in \mathbb{R} \mid 0 < A \leq 12150 \}$$

Example 2

Determine the domain and range of each function:



a) $f(x) = -2(x + 3)^2 - 8$



Domain = $\{x \in \mathbb{R}\}$

Range = $\{y \in \mathbb{R} \mid y \leq -8\}$

b) $g(x) = \sqrt{x + 3}$

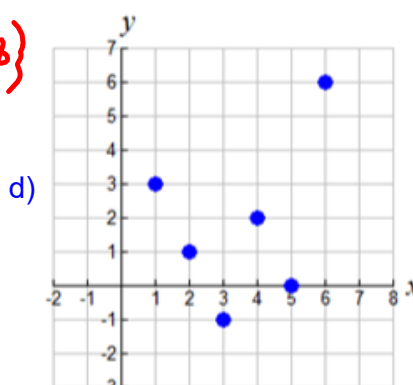
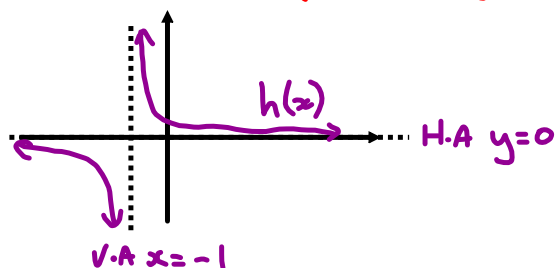
Domain = $\{x \in \mathbb{R} \mid x \geq -3\}$

Range = $\{y \in \mathbb{R} \mid y \geq 0\}$

c) $h(x) = 1/(x+1)$

Domain = $\{x \in \mathbb{R} \mid x \neq -1\}$

Range = $\{y \in \mathbb{R} \mid y \neq 0\}$



Domain = $\{x \in \mathbb{I} \mid 1 \leq x \leq 6\}$

set of integers

or Domain = $\{1, 2, 3, 4, 5, 6\}$

Range = $\{y \in \mathbb{I} \mid -1 \leq y \leq 6, y \neq 4, 5\}$

or Range = $\{-1, 0, 1, 2, 3, 6\}$

Homework: p 35-37 #2-5, 7, 9, 11-13, 16

(use the graphing technology, 'Desmos', for questions 9, 11 and 12 - later on in the unit you will need to do these questions without technology)

Factoring Examples

$$\begin{aligned} \textcircled{5} \text{ j) } & 49mn - 14m^2n^2 \\ & = \underline{\underline{7mn(7 - 2mn)}} \end{aligned}$$

$$\begin{aligned} \textcircled{7} \text{ d) } & a(2a-3b) - b(2a-3b) \\ & = (2a-3b)(a-b) \end{aligned}$$

$$\begin{aligned} \textcircled{6} \text{ a) } & x^4 - 5x^3 + 3x^2 \\ & = \underline{\underline{x^2(x^2 - 5x + 3)}} \end{aligned}$$

$$\begin{aligned} \textcircled{9} \text{ g) } & 2mx + 4x + 2my + 4y \\ & = 2x(m+2) + 2y(m+2) \\ & = (m+2)(2x+2y) \\ & = 2(m+2)(x+y) \end{aligned}$$

oops! Should have common factored first!

$$\begin{aligned} & \rightarrow = 2(mx + 2x + my + 2y) \\ & = 2[x(m+2) + y(m+2)] \\ & = 2[(m+2)(x+y)] \\ & = \underline{\underline{2(m+2)(x+y)}} \end{aligned}$$

$$\begin{aligned}
 \textcircled{4} \text{ j) } & 56 - 15x + x^2 \\
 & = x^2 - 15x + 56 \\
 & = \underline{\underline{(x-7)(x-8)}}
 \end{aligned}$$

$$\begin{aligned}
 -7 \times -8 & = 56 \\
 -7 + -8 & = -15
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{6} \text{ h) } & 16y^2 - 25x^2 \\
 & = \underline{\underline{(4y+5x)(4y-5x)}}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{5} \text{ l) } & 9m^2 - 30mn + 25n^2 \\
 & = \underline{\underline{(3m-5n)^2}}
 \end{aligned}$$

$2\sqrt{9m^2}\sqrt{25n^2}$
 $= \pm 2(3m)(5n)$
 $= \pm 30mn$

$$\begin{aligned}
 \textcircled{6} \text{ j) } & 3m^2 + 13m + 4 \\
 & = 3m^2 + 12m + m + 4 \\
 & = 3m(m+4) + 1(m+4) \\
 & = \underline{\underline{(m+4)(3m+1)}}
 \end{aligned}$$

$12 \times 1 = (3)(4)$
 $12 + 1 = 13$

$\rightarrow \underline{\underline{(m+4)(3m+1)}}$