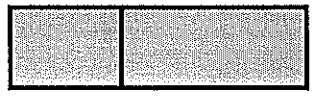
1.4 Domain and Range of a Function

Example 1

a) A farmer has 540m of fencing to enclose a rectangular area and divide it into two sections as shown:



a) Write an equation to express the total area enclosed as a function of the width.

b) Determine the domain and range of this area function.

Example 2

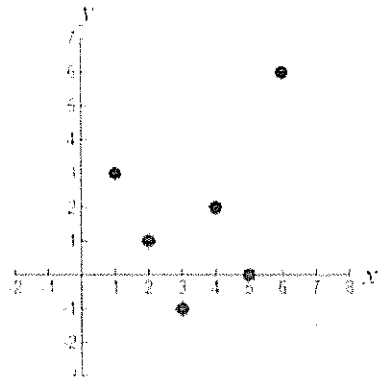
Determine the domain and range of each function:

a) $f(x) = -2(x + 3)^2 - 8$

b) $g(x) = \sqrt{(x + 3)}$

c) $h(x) = 1/(x+1)$

d)



Homework: p 35-37 #2-5, 7, 9, 11-13, 16

(use the graphing technology, 'Desmos', for questions 9, 11 and 12 - later on in the unit you will need to do these questions without technology)

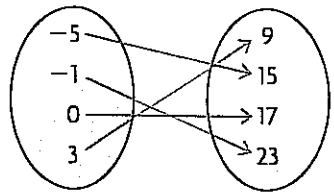
CHECK Your Understanding

1. State the domain and range of each relation.

a)

Year of Birth	Life Expectancy (years)
1900	47.3
1920	54.1
1940	62.9
1960	69.7
1980	73.7
2000	77.0

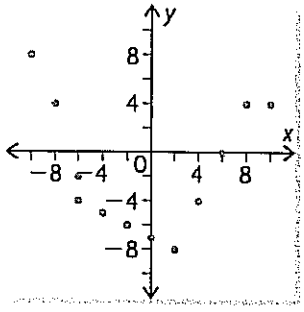
b)



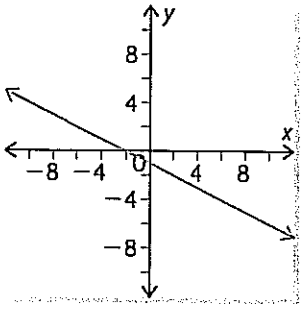
c) $\{(-4, 7), (0, 5), (0, 3), (3, 0), (5, -1)\}$

2. State the domain and range of each relation.

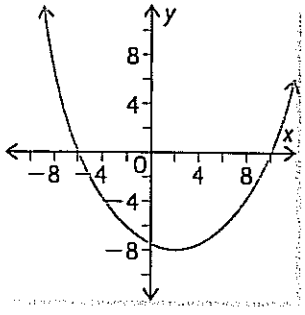
a)



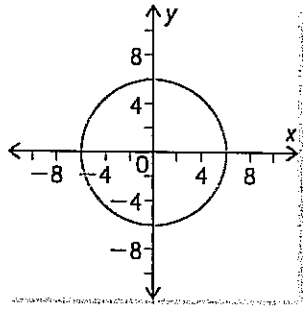
b)



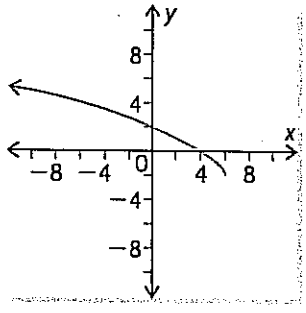
c)



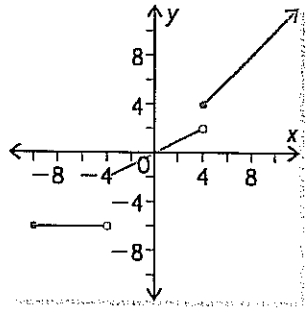
d)



e)



f)

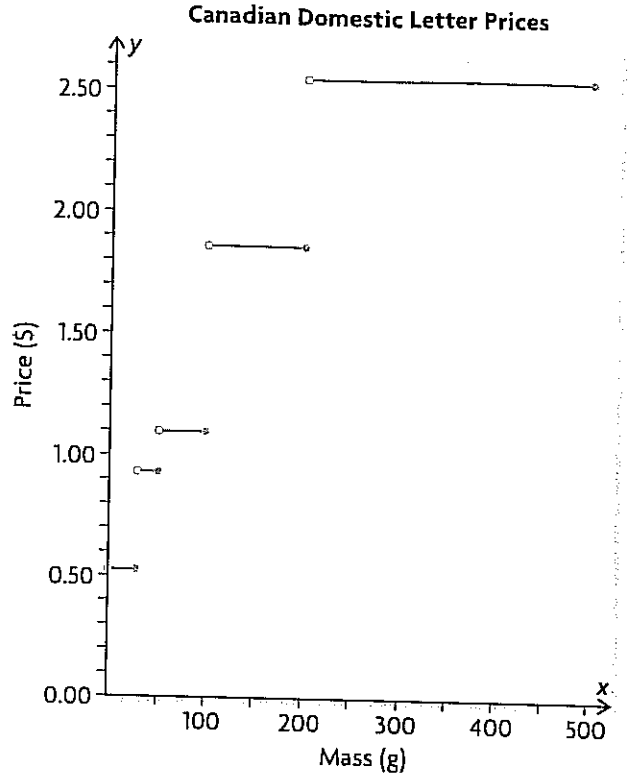


3. Identify which of the relations in questions 1 and 2 are functions.

4. Determine the domain and range of the function $f(x) = 2(x - 1)^2 - 3$ by sketching its graph.

PRACTISING

5. The graph shows how 2007 prices for mailing letters in Canada vary with mass.



a) Explain why this relation is a function. Why is it important for this to be so?

b) State the domain and range of the function.

6. The route for a marathon is 15 km long. Participants may walk, jog, run, or cycle. Copy and complete the table to show times for completing the marathon at different speeds.

Speed (km/h)	1	2	3	4	5	6	8	10	15	20
Time (h)	15.0	7.5								

Graph the relation in the table and explain how you know that it is a function. State the domain and range of the function.

7. A relation is defined by $x^2 + y^2 = 36$.

K a) Graph the relation.

b) State the domain and range of the relation.

c) Is the relation a function? Explain.

8. Write a function to describe coffee dripping into a 10-cup carafe at a rate of 1 mL/s. State the domain and range of the function (1 cup = 250 mL).

9. Determine the domain and range of each function.

a) $f(x) = -3x + 8$

b) $g(x) = -0.5(x + 3)^2 + 4$

c) $h(x) = \sqrt{x - 1}$

d) $p(x) = \frac{2}{3}(x - 2)^2 - 5$

e) $q(x) = 11 - \frac{5}{2}x$

f) $r(x) = \sqrt{5 - x}$

10. A ball is thrown upward from the roof of a 25 m building. The ball reaches a height of 45 m above the ground after 2 s and hits the ground 5 s after being thrown. A

a) Sketch a graph that shows the height of the ball as a function of time.

b) State the domain and range of the function.

c) Determine an equation for the function.

11. Write the domain and range of each function in set notation.

a) $f(x) = 4x + 1$

b) $f(x) = \sqrt{x - 2}$

c) $f(x) = 3(x + 1)^2 - 4$

d) $f(x) = -2x^2 - 5$

12. Use a graphing calculator to graph each function and determine the domain and range.

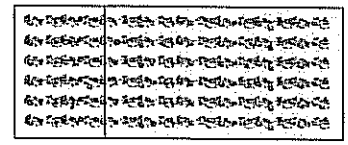
a) $f(x) = \sqrt{3 - x} + 2$

b) $g(x) = x^2 - 3x$

c) $h(x) = \frac{1}{x^2}$

d) $p(x) = \sqrt{x^2 - 5}$

13. A farmer has 450 m of fencing to enclose a rectangular area and divide it into two sections as shown. T



a) Write an equation to express the total area enclosed as a function of the width.

b) Determine the domain and range of this area function.

c) Determine the dimensions that give the maximum area.

14. Determine the range of each function if the domain is $\{-3, -1, 0, 2.5, 6\}$.

a) $f(x) = 4 - 3x$

b) $f(x) = 2x^2 - 3x + 1$

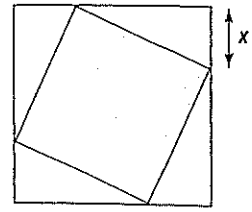
15. Explain the terms "domain" and "range" as they apply to relations and functions. Describe, with examples, how the domain and range are determined from a table of values, a graph, and an equation. C

Extending

16. a) Sketch the graph of a function whose domain is $\{x \in \mathbf{R}\}$ and range is $\{y \in \mathbf{R} \mid y \leq 2)\}$.

b) Sketch the graph of a relation that is not a function and whose domain is $\{x \in \mathbf{R} \mid x \geq -4\}$ and range is $\{y \in \mathbf{R}\}$.

17. You can draw a square inside another square by placing each vertex of the inner square on one side of the outer square. The large square in the diagram has side length 10 units.



a) Determine the area of the inscribed square as a function of x.

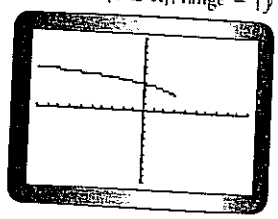
b) Determine the domain and range of this area function.

c) Determine the perimeter of the inscribed square as a function of x.

d) Determine the domain and range of this perimeter function.

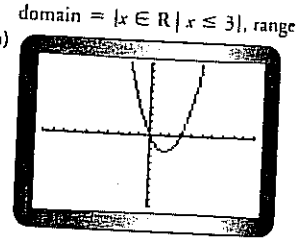
Lesson 1.4, pp. 35–37

1. a) domain = {1900, 1920, 1940, 1960, 1980, 2000},
   range = {47.3, 54.1, 62.9, 69.7, 73.7, 77.0}
   b) domain = {−5, −1, 0, 3}, range = {9, 15, 17, 23}
   c) domain = {−4, 0, 3, 5}, range = {−1, 0, 3, 5, 7}
2. a) domain = {0, ±2, ±4, ±6, ±8, ±10},
   range = {−8, −7, −6, −5, −4, −2, 0, 4, 8}
   b) domain = {$x \in R$}, range = {$y \in R$}
   c) domain = {$x \in R$}, range = {$y \in R \mid y \geq -8$}
   c) domain = {$x \in R$}, range = {$y \in R \mid y \geq -4$}
   d) domain = {$x \in R$}, range = {$y \in R \mid y \leq -5$}
12. a)



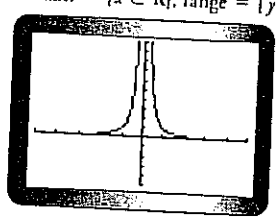
domain = {$x \in R \mid x \leq 3$}, range = {$y \in R \mid y \geq 2$}

b)



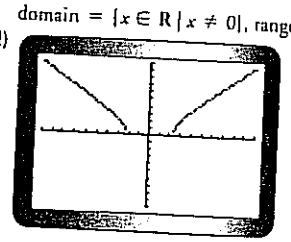
domain = {$x \in R$}, range = {$y \in R \mid y \geq -2.25$}

c)



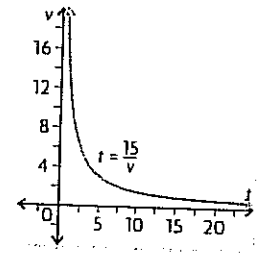
domain = {$x \in R \mid x \neq 0$}, range = {$y \in R \mid y > 0$}

d)



domain = {$x \in R \mid x \leq -\sqrt{5}, x \geq \sqrt{5}$},
range = {$y \in R \mid y \geq 0$}

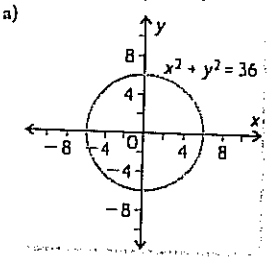
13. a) $A = \left(\frac{450 - 3w}{2}\right)w$
   b) domain = {$w \in R \mid 0 < w < 150$},
   range = {$A \in R \mid 0 < A \leq 8437.5$}
   c) $l = 112.5$ m, $w = 75$ m
   d) domain = {$x \in R \mid -6 \leq x \leq 6$},
   range = {$y \in R \mid -6 \leq y \leq 6$}
   e) domain = {$x \in R \mid x \leq 6$}, range = {$y \in R \mid y \geq -2$}
   f) domain = {$x \in R \mid x \geq -10$},
   range = {$y \in R \mid y = -6, -2 \leq y < 2, y \geq 4$}
3. 1. (a), (b); 2. (b), (c), (e), (f)
4. domain = {$x \in R$}, range = {$y \in R \mid y \geq -3$}
5. a) Even at masses when the price changes, a single price (the lower one) is assigned. It would not make sense to assign two or more prices to the same mass.
   b) domain = {$x \in R \mid 0 < x \leq 500$},
   range = {0.52, 0.93, 1.20, 1.86, 2.55}
6.



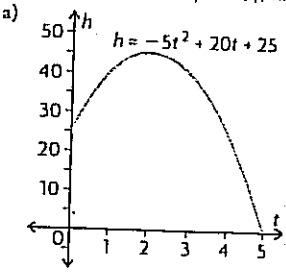
Graph passes vertical line test;
domain = {$v \in R \mid v > 0$},
range = {$t \in R \mid t > 0$}

Speed (km/h)	Time (h)
1	15.0
2	7.5
3	5.0
4	3.75
5	3.0
6	2.5
8	1.875
10	1.5
15	1.0
20	0.75

7. a)



   b) domain = {$x \in R \mid -6 \leq x \leq 6$},
   range = {$y \in R \mid -6 \leq y \leq 6$}
   c) No; fails vertical line test
8. $V(t) = t$; domain = {$t \in R \mid 0 \leq t \leq 2500$},
   range = {$V \in R \mid 0 \leq V \leq 2500$}
9. a) domain = {$x \in R$}, range = {$y \in R$}
   b) domain = {$x \in R$}, range = {$y \in R \mid y \leq 4$}
   c) domain = {$x \in R \mid x \geq 1$}, range = {$y \in R \mid y \geq 0$}
   d) domain = {$x \in R$}, range = {$y \in R \mid y \geq -5$}
   e) domain = {$x \in R$}, range = {$y \in R$}
   f) domain = {$x \in R \mid x \leq 5$}, range = {$y \in R \mid y \geq 0$}
10. a)



   b) domain = {$t \in R \mid 0 \leq t \leq 5$}, range = {$h \in R \mid 0 \leq h \leq 45$}
   c) $h = -5t^2 + 20t + 25$
11. a) domain = {$x \in R$}, range = {$y \in R$}
   b) domain = {$x \in R \mid x \geq 2$}, range = {$y \in R \mid y \geq 0$}