1.5 Inverse of Functions - Practice (complete circled questions)

1. Given the ordered pairs of each function, find the inverse, and graph the function and its inverse.

a) $f = \{(0, 2), (1, 3), (2, 4), (3, 5)\}$

b) $g = \{(-1, -3), (1, -2), (3, 4), (5, 0), (6, 1)\}$

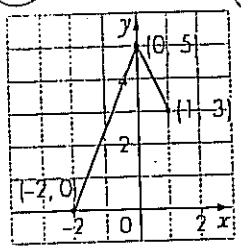
2. Given the ordered pairs of each function, find the inverse, and state whether the inverse is a function.

a) $f = \{(-2, 3), (-1, 2), (0, 0), (4, -2)\}$

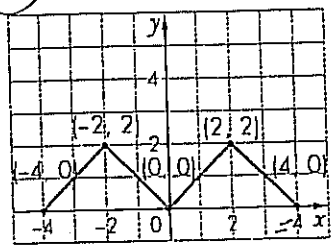
b) $g = \{(4, -2), (2, 1), (1, 3), (0, -2), (-3, -3)\}$

3. Sketch the inverse of each function.

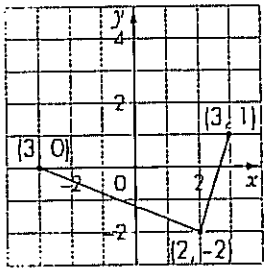
a)



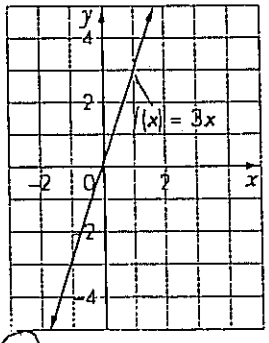
b)



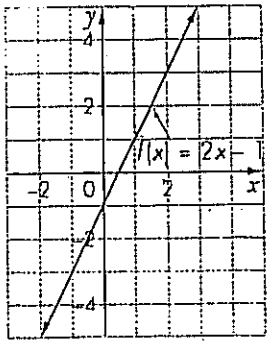
c)



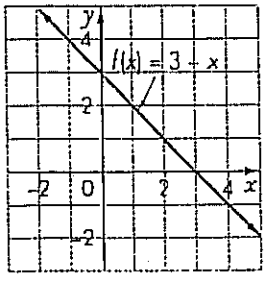
d)



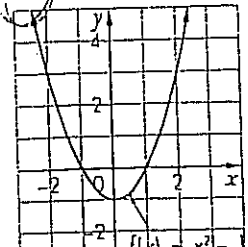
e)



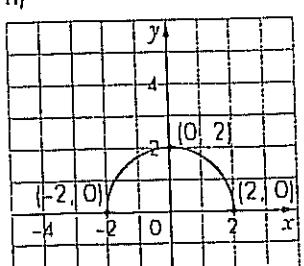
f)



g)



h)



5. Find the inverse of each function.

a) $f(x) = x - 1$ b) $f(x) = \frac{x}{2}$

c) $f(x) = x + 3$ d) $f(x) = \frac{4}{3}x$

e) $f(x) = 2x + 1$ f) $f(x) = \frac{x + 2}{3}$

g) $g(x) = \frac{5}{2}x - 4$ h) $h(x) = 0.2x + 1$

6. Find the inverse of each function. Graph the function and its inverse.

a) $f(x) = x + 2$ b) $f(x) = 4x$

c) $f(x) = 3x - 2$ d) $f(x) = x$

e) $f(x) = 3 - x$ f) $f(x) = \frac{x - 2}{3}$

7. Find the inverse of each function and determine whether the inverse is a function.

a) $f(x) = 2x - 5$ b) $f(x) = \frac{x + 3}{4}$

c) $f(x) = \frac{x}{4} + 3$ d) $f(x) = 5 - x$

8. Determine if the functions in each pair are inverses of each other.

a) $f(x) = x + 5$ and $g(x) = x - 5$

b) $f(x) = 7x$ and $g(x) = \frac{x}{7}$

c) $f(x) = 2x - 1$ and $g(x) = \frac{x + 1}{2}$

d) $f(x) = x - 3$ and $g(x) = 3 - x$

e) $f(x) = \frac{x}{3} - 4$ and $g(x) = 3x - 4$

f) $g(x) = \frac{x}{3} - 5$ and $h(x) = 3x + 5$

g) $h(x) = \frac{x - 8}{4}$ and $k(x) = 4(x + 2)$

9. Verify algebraically and graphically that the functions in each pair are inverses of each other.

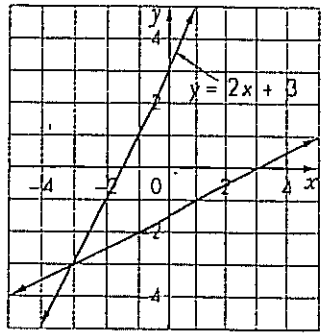
a) $y = 3x + 4$ and $y = \frac{1}{3}(x - 4)$

b) $y = 3 - 2x$ and $y = -\frac{1}{2}(x - 3)$

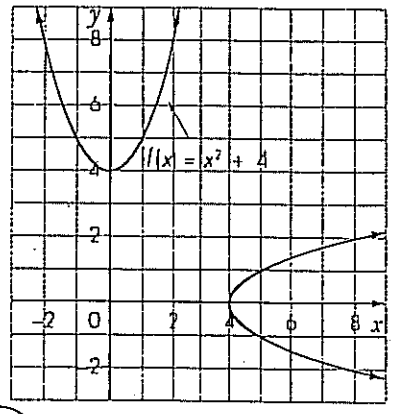
10. For each of the following functions,
a) find the inverse of $f(x)$
b) graph $f(x)$ and its inverse
c) determine the domain and range of $f(x)$ and its inverse
i) $f(x) = x^2 - 3$
ii) $f(x) = x^2 + 1$
iii) $f(x) = -x^2$
iv) $f(x) = -x^2 - 1$
v) $f(x) = (x - 2)^2$
vi) $f(x) = (x + 1)^2$

11. The blue graph is a reflection of the red graph in the line $y = x$. The equation of the red graph is given. Write the equation of the blue graph.

a)



b)



12. Determine if the functions in each pair are inverses of each other.
a) $y = x^2 - 3$ and $y = \sqrt{x + 3}$
b) $y = x^2 + 1$ and $y = \sqrt{x + 1}$

13. Find the inverse of each function. If the inverse is a function, determine the domain and range of the inverse.

a) $y = 2x - 3$
b) $y = 2 - 4x$
c) $y = 3(x - 2)$
d) $y = \frac{1}{2}(x - 6)$
e) $y = x^2$
f) $y = x^2 + 2$
g) $y = x^2 - 4$
h) $y = 2x^2 - 1$
i) $y = (x - 3)^2$
j) $y = (x + 2)^2$

15. Find the inverse of each of the following functions.
a) $y = \sqrt{x - 2}$
b) $y = \sqrt{3 - x}$
c) $y = \sqrt{x^2 + 9}$

Apply, Solve, Communicate

19. **Van Rental** The cost of renting a van for one day is a flat rate of $50, plus a variable rate of $0.15/km.
a) Write a function to express the total cost of a one-day rental, $c(d)$ dollars, in terms of the distance driven, d kilometres.
b) Determine the inverse of the function.
c) What does the inverse represent?
d) What is the domain of the inverse?

B

20. **Measurement** a) Let x represent the radius of a circle. Write a function $f(x)$ to express the circumference in terms of the radius.
b) Find the inverse of this function.
c) Is the inverse a function?
d) What does the inverse represent?

21. **Application** a) Let x represent the radius of a sphere. Write a function $f(x)$ to express the surface area in terms of the radius.
b) Find the inverse of this function. & graph $f(x)$ and $f^{-1}(x)$.
c) Determine the domain and range of the inverse.
d) Is the inverse a function?
e) What does the inverse represent?

22. **Retail Sales** A sale at an appliance store advertised that all appliances were being sold at 30% off the original selling price.
a) Write a function that gives the sale price as a function of the original selling price.
b) Find the inverse of this function.
c) What does the inverse represent?

23. **Foreign currency exchange** One day, the Canadian dollar was worth US$0.70.
a) Write a function that expresses the value of the US dollar, u, in terms of the Canadian dollar, c.
b) Find the inverse. Round the coefficient to the nearest hundredth. Graph both.
c) Use the inverse to convert US$150 to Canadian dollars.

24. **Geology** The approximate temperature, in degrees Celsius, of rocks beneath the surface of the Earth can be found by multiplying their depth, in kilometres, by 35 and adding 20 to the product.

a) Let d kilometres represent the depth of some rocks. Write a function $T(d)$ that expresses the Celsius temperature of the rocks in terms of their depth.

b) Write the inverse function.

c) At what depth do rocks have a temperature of 90°C?

25. **Weekly wages** Jana works at a clothing store. She earns $400 a week, plus a commission of 5% of her sales.

a) Write a function that describes Jana's total weekly earnings as a function of her sales.

b) Find the inverse of this function. Graph both

c) What does the inverse represent?

d) One week, Jana earned $575. Calculate her sales that week.

26. **Measurement** The measure of an interior angle, i, of a regular polygon is related to the number of sides n by the function

$$i(n) = 180 - \frac{360}{n}.$$

a) Determine the measure of an interior angle of a regular heptagon.

b) Find the inverse of the function. Graph both.

c) Use the inverse to identify the regular polygon with interior angles of 144°.

27. **Falling objects** If an object is dropped from a height of 80 m, its approximate height, $h(t)$ metres, above the ground t seconds after being dropped is given by the function $h(t) = -5t^2 + 80$.

a) Graph the function.

b) Find and graph the inverse.

c) Is the inverse a function? Explain.

d) What does the inverse represent?

e) After what length of time is the object 35 m above the ground?

f) How long does the object take to reach the ground?

28. a) Given $f(x) = 2x - 4$, write equations for $-f(x)$, $f(-x)$, and $f^{-1}(x)$.

b) Sketch the four graphs on the same set of axes.

c) Determine any points that are invariant for each reflection.

29. a) Given $f(x) = -3x + 2$, write equations for $-f(x)$, $f(-x)$, and $f^{-1}(x)$.

b) Sketch the four graphs on the same set of axes.

c) Determine any points that are invariant for each reflection.

30. a) Given $f(x) = \sqrt{x+3}$, write equations for $-f(x)$, $f(-x)$, and $f^{-1}(x)$.

b) Sketch the four graphs on the same set of axes.

Answers

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1. a) $f^{-1} = \{(2, 0), (3, 1), (4, 2), (5, 3)\}$ b) $g^{-1} = \{(-3, -1), (-2, 1), (4, 3), (0, 5), (1, 6)\}$ 2. a) $f^{-1} = \{(3, -2), (2, -1), (0, 0), (-2, 4)\}$; a function b) $g^{-1} = \{(-2, 4), (1, 2), (3, 1), (-2, 0), (-3, -3)\}$; not a function 4. a) $x = \dfrac{f(x) - 2}{3}$

b) $x = \dfrac{12 - 3f(x)}{2}$ c) $x = \dfrac{3 - f(x)}{4}$ d) $x = 4f(x) - 3$

e) $x = 2f(x) + 10$ f) $x = \pm\sqrt{y - 3}$ 5. a) $f^{-1}(x) = x + 1$

b) $f^{-1}(x) = 2x$ c) $f^{-1}(x) = x - 3$ d) $f^{-1}(x) = \dfrac{3}{4}x$

e) $f^{-1}(x) = \dfrac{x - 1}{2}$ f) $f^{-1}(x) = 3x - 2$ g) $g^{-1}(x) = \dfrac{2x + 8}{5}$

h) $h^{-1}(x) = 5x - 5$ 6. a) $f^{-1}(x) = x - 2$ b) $f^{-1}(x) = \dfrac{x}{4}$

c) $f^{-1}(x) = \dfrac{x + 2}{3}$ d) $f^{-1}(x) = x$ e) $f^{-1}(x) = 3 - x$

f) $f^{-1}(x) = 3x + 2$ 7. a) $f^{-1}(x) = \dfrac{x + 5}{2}$, function

b) $f^{-1}(x) = 4x - 3$, function c) $f^{-1}(x) = 4x - 12$, function d) $f^{-1}(x) = 5 - x$, function 8. a) yes b) yes c) yes d) no e) no f) no g) yes 9. a) reflections in the line $y = x$ b) reflections in the line $y = x$ 10. i) a) $f^{-1}(x) = \pm\sqrt{x + 3}$ c) $f(x)$: domain: all real numbers, range: $y \geq -3$; $f^{-1}(x)$: domain: $x \geq -3$, range: all real numbers ii) a) $f^{-1}(x) = \pm\sqrt{x - 1}$ c) $f(x)$: domain: all real numbers, range: $y \geq 1$; $f^{-1}(x)$: domain: $x \geq 1$, range: all real numbers iii) a) $f^{-1}(x) = \pm\sqrt{-x}$ c) $f(x)$: domain: all real numbers, range: $y \leq 0$; $f^{-1}(x)$: domain $x \leq 0$, range: all real numbers iv) a) $f^{-1}(x) = \pm\sqrt{-x - 1}$ c) $f(x)$: domain: all real numbers, range $y \leq -1$; $f^{-1}(x)$: domain: $x \leq -1$, range: all real numbers v) a) $f^{-1}(x) = \pm\sqrt{x + 2}$ c) $f(x)$: domain: all real numbers, range: $y \geq 0$; $f^{-1}(x)$: domain: $x \geq 0$, range: all real numbers vi) a) $f^{-1}(x) = \pm\sqrt{x - 1}$ c) $f(x)$: domain: all real numbers, range: $y \geq 0$; $f^{-1}(x)$ = domain: $x \geq 0$, range: all real numbers 11. a) $y = \dfrac{x - 3}{2}$ b) $y = \pm\sqrt{x - 4}$ 12. a) no

b) no 13. a) $f^{-1}(x) = \dfrac{x + 3}{2}$, domain: all real numbers, range: all real numbers b) $f^{-1}(x) = \dfrac{2 - x}{4}$, domain: all real numbers, range: all real numbers c) $f^{-1}(x) = \dfrac{x + 6}{3}$, domain: all real numbers, range: all real numbers d) $f^{-1}(x) = 2x + 6$, domain: all real numbers, range: all real numbers

e) $f^{-1}(x) = \pm\sqrt{x}$ f) $f^{-1}(x) = \pm\sqrt{x - 2}$ g) $f^{-1}(x) = \pm\sqrt{x + 4}$

h) $f^{-1}(x) = \pm\dfrac{\sqrt{2x + 2}}{2}$ i) $f^{-1}(x) = \pm\sqrt{x} + 3$ j) $f^{-1}(x) = \pm\sqrt{x} - 2$

14. i) a) $f^{-1}(x) = \sqrt{x}$ c) $f(x)$: domain: $x \geq 0$, range: $y \geq 0$; $f^{-1}(x)$: domain: $x \geq 0$, range: $y \geq 0$ ii) a) $f^{-1}(x) = \sqrt{x + 2}$ c) $f(x)$: domain: $x \geq 0$, range: $y \geq -2$; $f^{-1}(x)$: domain: $x \geq -2$, range: $y \geq 0$ iii) a) $f^{-1}(x) = -\sqrt{x - 4}$ c) $f(x)$: domain: $x \leq 0$, range: $y \geq 4$; $f^{-1}(x)$: domain: $x \geq 4$, range: $y \leq 0$ iv) a) $f^{-1}(x) = \sqrt{3 - x}$ c) $f(x)$: domain: $x \geq 0$, range: $y \leq 3$; $f^{-1}(x)$: domain: $x \leq 3$, range: $y \geq 0$ v) a) $f^{-1}(x) = \sqrt{x + 4}$ c) $f(x)$: domain: $x \geq 4$, range: $y \geq 0$; $f^{-1}(x)$: domain: $x \geq 0$, range: $y \geq 4$ vi) a) $f^{-1}(x) = -\sqrt{x - 3}$ c) $f(x)$: domain: $x \leq -3$, range: $y \geq 0$; $f^{-1}(x)$: domain: $x \geq 0$, range: $y \leq -3$

15. a) $f^{-1}(x) = x^2 + 2$, $x \geq 0$ b) $f^{-1}(x) = 3 - x^2$, $x \geq 0$

c) $f^{-1}(x) = \pm\sqrt{x^2 - 9}$, $x \geq 3$ 16. i) a) $f^{-1}(x) = \pm\sqrt{x - 3}$

c) domain: $x \geq 0$ ii) a) $f^{-1}(x) = \pm\dfrac{\sqrt{2x}}{2}$ c) domain: $x \geq 0$

iii) a) $f^{-1}(x) = \pm\sqrt{x + 1}$ c) domain: $x \geq 0$ iv) a) $f^{-1}(x) = \pm\sqrt{-x}$ c) domain: $x \geq 0$ v) a) $f^{-1}(x) = \pm\sqrt{1 - x}$ c) $x \geq 0$ vii) a) $f^{-1}(x) = \pm\sqrt{x} + 2$ c) $x \geq 2$ vii) a) $f^{-1}(x) = \pm\sqrt{x} + 4$ c) $x \geq 4$ viii) a) $f^{-1}(x) = \pm\sqrt{-x} - 5$ c) $x \geq -5$ 17. a) $f^{-1}(x) = \dfrac{1}{x}$ b) Yes; for every value of x there is only one corresponding value of y. 18. a) $f^{-1}(x) = x^2$, $x \geq 0$ b) yes 19. a) $c(d) = 50 + 0.15d$

b) $d(c) = \dfrac{c - 50}{0.15}$ c) The inverse represents the distance that can be driven for a given rental cost. d) $c \geq 50$

20. a) $f(x) = 2\pi x$, $x \geq 0$ b) $f^{-1}(x) = \dfrac{x}{2\pi}$ c) yes

d) The inverse represents the length of the radius for a circle of a given circumference. 21. a) $f(x) = 4\pi x^2$, $x \geq 0$

b) $f^{-1}(x) = \dfrac{\sqrt{\pi x}}{2\pi}$ c) domain: $x \geq 0$, range: $y \geq 0$ d) yes e) The inverse represents the length of the radius for a sphere of a given surface area. 22. a) $f(x) = 0.7x$ b) $f^{-1}(x) = \dfrac{x}{0.7}$ c) The inverse represents the original selling price as a function of the sale price. 23. a) $u(c) = 0.7c$ b) $c(u) = 1.43u$ c) \$214.50

24. a) $T(d) = 35d + 20$ b) $d(T) = \dfrac{T - 20}{35}$ c) 2 km

25. a) $E(s) = 400 + 0.05s$ b) $s(E) = 20E - 8000$ c) The inverse represents the amount of sales as a function of earnings.

d) \$3500 26. a) 128.6° b) $n(i) = \dfrac{360}{180 - i}$ c) decagon

27. b) $t(h) = \dfrac{\sqrt{400 - 5h}}{5}$ c) yes, since the domain of h is restricted to $t \geq 0$ d) The inverse represents the time for an object to fall from a height of 80 m to h metres above the ground. e) 3 s f) 4 s 28. a) $-f(x) = -2x + 4$, $f(-x) = -2x - 4$, $f^{-1}(x) = \dfrac{x + 4}{2}$ c) $-f(x)$: $(2, 0)$, $f(-x)$: $(0, -4)$, $f^{-1}(x)$: $(4, 4)$ 29. a) $-f(x) = 3x - 2$, $f(-x) = 3x + 2$, $f^{-1}(x) = \dfrac{2 - x}{3}$ c) $-f(x)$: $\left(\dfrac{2}{3}, 0\right)$, $f(-x)$: $(0, 2)$, $f^{-1}(x)$: $\left(\dfrac{1}{2}, \dfrac{1}{2}\right)$ 30. a) $-f(x) = -\sqrt{x + 3}$, $f(-x) = \sqrt{3 - x}$, $f^{-1}(x) = x^2 - 3$, $x \geq 0$ 31. $y = 2$ 33. Answers