

1.5 Inverse of a Function

In this section, you will learn about the inverse of a function and how it relates to the idea of reversing operations.

Inverse of a function:

- The inverse of a function f is denoted by f^{-1} .
- The function and its inverse have the property that if $f(a) = b$, then $f^{-1}(b) = a$.

The notation $f^{-1}(x)$ is read as “the inverse of f at x .”

Note that the -1 in $f^{-1}(x)$ does not behave like an exponent, so $f^{-1}(x) \neq \frac{1}{f(x)}$.

Methods of finding the inverse:

1. If you are given a data table with points of a given function (x, y) the inverse of the function can be plotted using they points given by switching the x and y values of each point. i.e. for any $f(x)$ each value $(x, y) \rightarrow (y, x)$ for its inverse $f^{-1}(x)$.
2. The inverse of a function $f(x)$ can be found by reversing the operations that the function specifies. Consider the function $f(x) = 3x + 2$. This function multiplies each x -value by 3 and adds 2 to the result. Reversing the operations then subtracts 2 from each x and divides the result by 3. So, the inverse of $f(x)$ is $f^{-1}(x) = \frac{x-2}{3}$. The inverse is the “reverse” of the original function.

A systematic method for determining the inverse of a function algebraically can be described by the following steps:

1. Write the equation in “ $y =$ ” form, if it is not already in that form.
2. Interchange x and y in the equation.
3. Solve the new equation for y .
4. Replace y with $f^{-1}(x)$.

The domain of $f(x)$ is the range of $f^{-1}(x)$, and the range of $f(x)$ is the domain of $f^{-1}(x)$.

Summary of Key Concepts

- The inverse of a function $f(x)$ is denoted by $f^{-1}(x)$.
- The inverse of a function can be found by interchanging the x - and y -coordinates of the function.
- The graph of $f^{-1}(x)$ is the graph of $f(x)$ reflected in the line $y = x$.
- The inverse of a function can be found by interchanging x and y in the equation of the function and then solving the new equation for y .
- For algebraic inverses of quadratic functions, the functions must be in vertex form.
- The inverse of a function is not necessarily a function.

Homework: p47, 48 #2, 4, 8, 9, 10