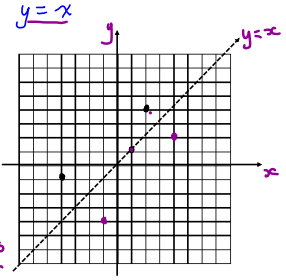
1.5 Inverses of Functions

- When a fxn is inversed, the x and y values change places because inverses are reflections in the line $y = x$



$f(x)$ → $f^{-1}(x)$

$(2,4) \rightarrow (4,2)$

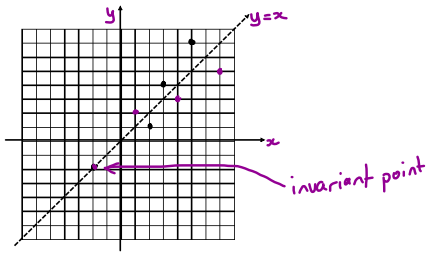
$(-4,-1) \rightarrow (-1,-4)$

$(1,1) \rightarrow (1,1)$

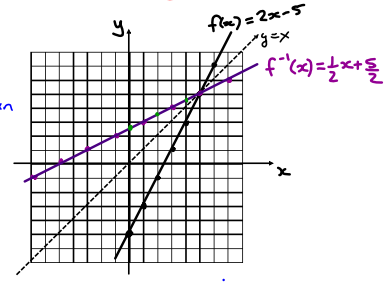
x and y components of each ordered pair have switched

eg) given $f(x) = \{(2,1)\ (3,4)\ (5,7)\ (-2,-2)\}$

write $f^{-1}(x) = \{(1,2)\ (4,3)\ (7,5)\ (-2,-2)\}$



eg 2. a) Graph the linear equation $y = 2x - 5$ and it's inverse



b) Determine an equation for the inverse of the fxn

$f(x) = 2x - 5$

$y = 2x - 5$

$f^{-1}(x) \rightarrow \quad x = 2y - 5$

communicate that you are calculating the equation of the function inverse

exchange 'y' for 'x' and 'x' for 'y'

Isolate for y → $2y - 5 = x$

$2y = x + 5$

$y = \frac{x+5}{2}$

$\therefore \quad \boxed{f^{-1}(x) = \frac{1}{2}x + \frac{5}{2}}$

Example 3 Determine an equation for the inverse of the function $f(x) = x^2 - 6$ $(y = x^2 - 6)$

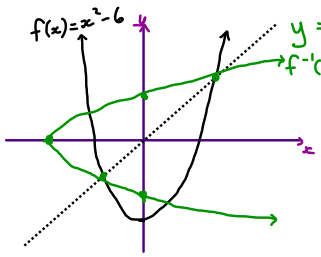
$f^{-1}(x) \rightarrow \quad x = y^2 - 6$

$y^2 - 6 = x$

$y^2 = x + 6$

$y = \pm\sqrt{x+6}$

$f^{-1}(x) = \pm\sqrt{x+6}$



Note The inverse of a function is not always described as a function because it may not pass the vertical line test.

12. The formula for converting a temperature in degrees Celsius into degrees Fahrenheit is $F = \frac{9}{5}C + 32$. Shirelle, an American visitor to Canada, uses a simpler rule to convert from Celsius to Fahrenheit: Double the Celsius temperature, then add 30.

A

a) Use function notation to write an equation for this rule. Call the function f and let x represent the temperature in degrees Celsius.

b) Write f^{-1} as a rule. Who might use this rule?

c) Determine $f^{-1}(x)$.

d) One day, the temperature was 14 °C. Use function notation to express this temperature in degrees Fahrenheit.

e) Another day, the temperature was 70 °F. Use function notation to express this temperature in degrees Celsius.

a) $f(x) = 2x + 30$

b) Subtract 30 then divide by 2

c) $f^{-1}(x) = \frac{x-30}{2}$

or $f^{-1}(x) = \frac{1}{2}x - 15$

$f^{-1}(x)$

$x = 2y + 30$

$2y + 30 = x$

$2y = x - 30$

$y = \frac{x-30}{2}$

$f^{-1}(x) = \frac{x-30}{2}$

d) $f(14) = 2(14) + 30$

$= 28 + 30$

$= 58$

∴ 14°C is approx 58F

e) $f^{-1}(70) = \frac{70-30}{2}$

$= \frac{40}{2}$

$= 20$

∴ 70F is approx 20°C