1.8 Continued - Combinations of Transformations

Homework take up

1. Describe how the graph of each of the following functions can be obtained from the graph of $y = f(x)$.

a) $y = 2f(x) + 3$ b) $y = \frac{1}{2}f(x) - 2$

c) $y = f(x+4) + 1$ d) $y = 3f(x-5)$

e) $y = f\left(\frac{1}{2}x\right) - 6$ f) $y = f(2x) + 1$

$$g(x) = a f[k(x-d)] + c$$

horizontally stretched/compressed by a factor of $\frac{1}{k}$

reflect through y-axis

vertically stretched/compressed by a factor of 'a'

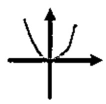
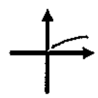
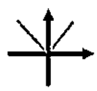
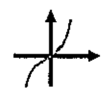
reflect through x axis $(a<0)$

translated horizontally d units

translated vertically c units

a)	Vertically Stretched by a factor of 2, translated 3 units up
b)	Vertically Compressed by a factor of $\frac{1}{2}$, translated 2 units down.
c)	Translated 4 units to the left and 1 unit up
d)	Vertically Stretched by a factor of 3, Translated 5 units to the right.
e)	Horizontally Stretched by a factor of 2, translated down 6 units.
f)	Horizontally compressed by a factor of $\frac{1}{2}$, translated 1 unit up.

Key Points - to be used to graph transformations of base functions

$y=x^2$	$y=\sqrt{x}$	$y=\lvert x\rvert$	$y=\frac{1}{x}$	$y=x^3$
(0,0) (1,1) (2,4) 	(0,0) (1,1) (4,2) 	(0,0) (1,1) (2,2) 	(1,1) (-1,-1) V.A. $x=0$ H.A. $y=0$ 	(-1,-1) (0,0) (1,1) 

Examples: Graph each base function and transformed function on the same set of axes.

a) $g(x) = 3f(x-1) - 2$ for $f(x) = x^2$

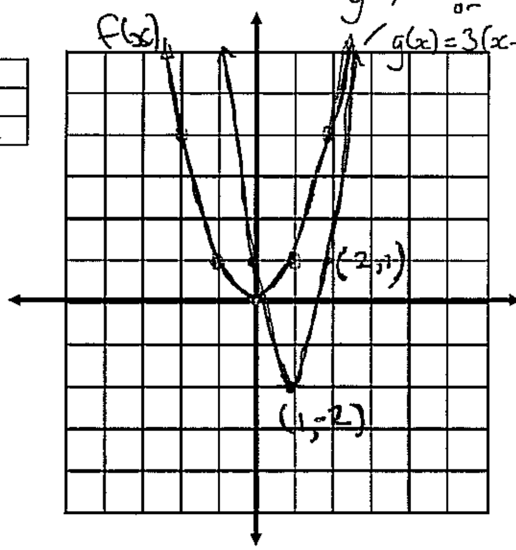
	x	y
R	x	x
S		3
T	1	-2

	x	y
R		
S	$\frac{1}{k}$	a
T	d	c

$g(x) = 3f(x-1) - 2$

or

$g(x) = 3(x-1)^2 - 2$



K.P. (f(x))	→	I.P. (g(x))
(0,0)	→	(1,-2)
(1,1)	→	(2,1)
(2,4)	→	(3,10)

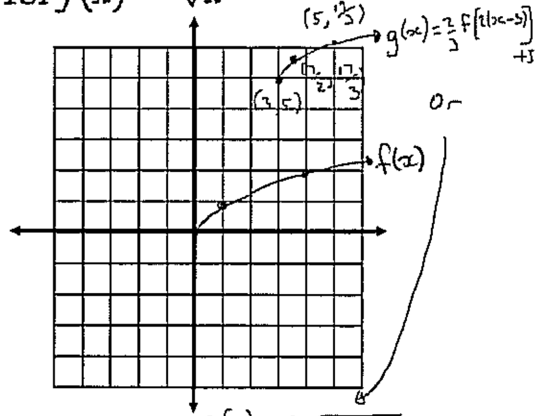
b) $g(x) = \frac{2}{3} f(2x-6) + 5$ for $f(x) = \sqrt{x}$

$g(x) = \frac{2}{3} f[2(x-3)] + 5$

'K' must be factored out!

R		
S	$\frac{1}{2}$	$\frac{2}{3}$
T	3	5

K.P. $(f(x))$	$\rightarrow$	I.P. $(g(x))$
$(0,0)$	$\rightarrow$	$(3,5)$
$(1,1)$	$\rightarrow$	$(\frac{7}{2}, \frac{17}{3})$
$(4,2)$	$\rightarrow$	$(5, \frac{19}{3})$



$g(x) = \frac{2}{3}\sqrt{2(x-3)} + 5$

c) $g(x) = -2f(3x+9) - 2$ for $f(x) = |x|$

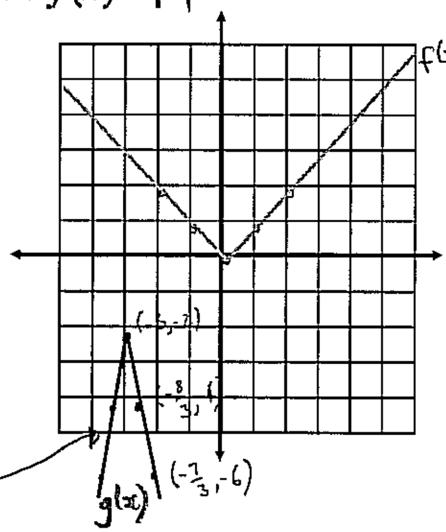
$g(x) = -2f[3(x+3)] - 2$

R	×	✓
S	$\frac{1}{3}$	-2
T	-3	-2

K.P $(f(x))$	$\rightarrow$	I.P $(g(x))$
$(0,0)$	$\rightarrow$	$(-3,-2)$
$(1,1)$	$\rightarrow$	$(-\frac{8}{3}, -4)$
$(2,2)$	$\rightarrow$	$(-\frac{7}{3}, -6)$

$g(x) = -2f[3(x+3)] - 2$

$g(x) = -2|3(x+3)| - 2$

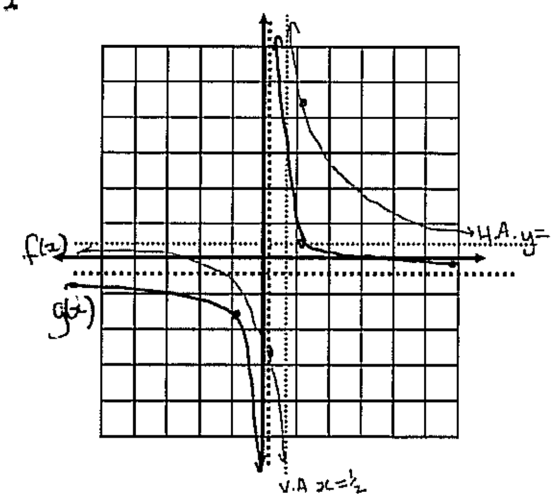


d) $g(x) = 4f(2x-1) + 1$ for $f(x) = \frac{1}{x}$

$g(x) = 4f[2(x-\frac{1}{2})] + 1$

R	×	×
S	$\frac{1}{2}$	4
T	$\frac{1}{2}$	1

K.P $(f(x))$	$\rightarrow$	I.P $(g(x))$
$(1,1)$	$\rightarrow$	$(1,5)$
$(-1,-1)$	$\rightarrow$	$(0,-3)$
V.A. $x=0$	$\rightarrow$	$x=\frac{1}{2}$
H.A. $y=0$	$\rightarrow$	$y=1$



$g(x) = 4f[2(x-\frac{1}{2})] + 1$

$g(x) = 4\frac{1}{2(x-\frac{1}{2})} + 1$

Homework / Classwork - complete #6 below, then p70 #4, 18, 22 (attached)

p70 #4, 18, 22

6. Sketch each set of functions on the same set of axes in the given order.

a) $y = x$
$y = 3x$
$y = 3(x-2)+10$

$g(x) = 3f(x-2)+10$

b) $y = x^2$
$y = 2x^2$
$y = 2(x+2)^2 - 3$

$g(x) = 2f(x+2) - 3$

c) $y = |x|$
$y = 0.5x$
$y = -0.5|x-4|+2$

$g(x) = -0.5f(x-4)+2$

d) $y = x^2$
$y = \left(\frac{1}{2}x^2\right)$
$y = \frac{1}{2}(x+3)^2+3$

e) $y = \sqrt{x}$
$y = \sqrt{2x}$
$y = -\sqrt{2x}$
$y = -\sqrt{2(x-1)} - 3$

$g(x) = -f[2(x-1)]-3$

f) ~~$y = \frac{1}{x}$~~
~~$y = \frac{2}{x}$~~
~~$y = \frac{2}{-x}$~~
~~$y = \frac{2}{-(x-3)} + 5$~~

$y = \frac{1}{x}$

$y = \frac{2}{-(x-3)} + 5$

$g(x) = 2f[-(x-3)]+5$

Open Book Assignment tomorrow

4. Explain what transformations you would need to apply to the graph of $y = f(x)$ to graph each function.

a) $y = 3f(x) - 1$

b) $y = f(x - 2) + 3$

c) $y = f(2x) - 5$

d) $y = -f\left(\frac{1}{2}x\right) - 2$

e) $y = \frac{2}{3}f(x + 3) + 1$

f) $y = 4f(-x) - 4$

18. Match each equation to its graph. Explain your reasoning.

a) $y = \dfrac{3}{-(x - 2)} + 1$

b) $y = 2|x - 3| - 2$

c) $y = -2\sqrt{x + 3} - 2$

d) $y = (0.25(x - 2))^2 - 3$

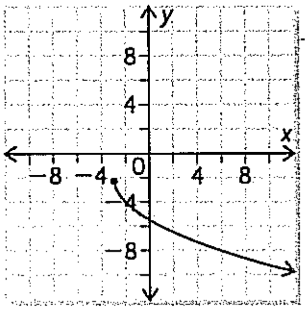
e) $y = -\dfrac{4}{x} - 3$

f) $y = -0.5|x + 4| + 2$

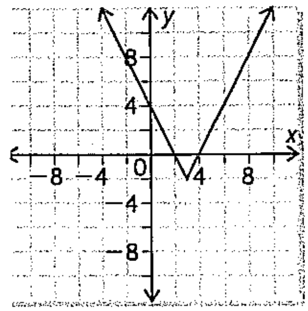
g) $y = -0.5\sqrt{1 - x} + 1$

h) $y = -\dfrac{1}{2}(x + 4)^2 + 1$

A



E



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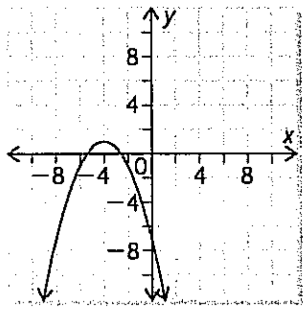
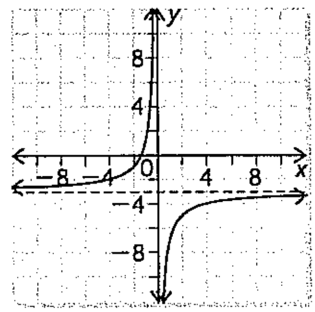


F



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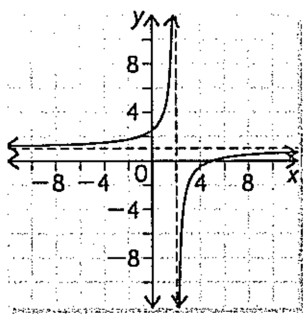

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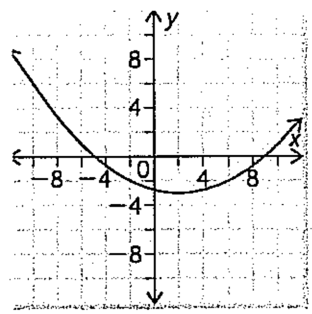
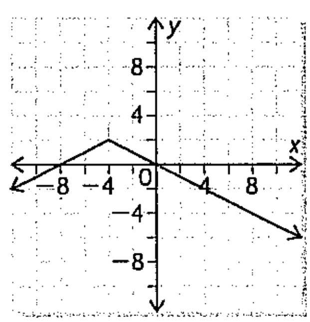
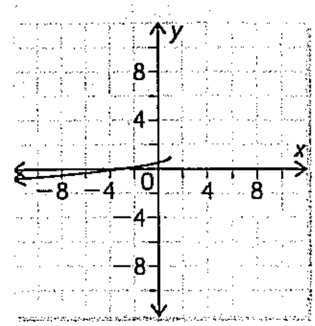


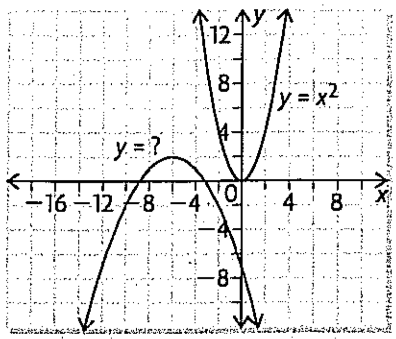
D



H



22. The graphs of $y = x^2$ and another parabola are shown.



a) Determine a combination of transformations that would produce the second parabola from the first.

b) Determine a possible equation for the second parabola.

Answers

4. a) Vertical stretch, factor 3, then translation 1 unit down

b) Translation 2 units right and 3 units up

c) Horizontal compression, factor $\frac{1}{2}$, then translation 5 units down

d) Reflection in x-axis, horizontal stretch with factor 2, and then translation 2 units down

e) Vertical compression, factor $\frac{2}{3}$, then translation 3 units left and 1 unit up

f) Vertical stretch with factor 4, reflection in y-axis, and then translation 4 units down

22. a) Reflection in x-axis, vertical compression factor $\frac{1}{4}$ [or horizontal stretch factor 2], and then translation 3 units left and 1 unit up

b) $y = -\frac{1}{4}(x + 6)^2 + 2$ $\left[\text{or } y = -\left[\frac{1}{2}(x + 6)\right]^2 + 2\right]$

18. a) C; parent graph is $y = \frac{1}{x}$, asymptotes are translated 2 units right and 1 unit up, and graph has been reflected in one of the axes

b) E; parent graph is $y = |x|$, and vertex is translated 3 units right and 2 units down

c) A: parent graph is $y = \sqrt{x}$, graph has been reflected in y-axis, and vertex is translated 3 units left and 2 units down

d) G: parent graph is $y = x^2$, and vertex is translated 2 units right and 3 units down

e) F; parent graph is $y = \frac{1}{x}$, asymptotes are translated 3 units down, and graph has been reflected in one of the axes

f) D; parent graph is $y = |x|$, graph has been reflected in y-axis, and vertex is translated 4 units left and 2 units up

g) H: parent graph is $y = \sqrt{x}$, graph has been reflected in x- and y-axes, and vertex is translated 1 unit right and 1 unit up

h) B: parent graph is $y = x^2$, graph has been reflected in y-axis, and vertex is translated 4 units left and 1 unit up