

2.4 Simplifying Rational Expressions

Rational expressions are those which have the form of a fraction such as where a and b are integers and b cannot equal zero.

$\frac{a}{b}$ such as $\frac{2}{3}$ or $\frac{17}{4}$; recall that $\frac{0}{3} = 0$ but $\frac{3}{0}$ is undefined.

In function notation, we would write $\frac{f(x)}{g(x)}$ where $g(x) \neq 0$

Because rational expressions are not defined when the denominator is zero, we must RESTRICT or set limits on any denominator containing a variable(s).

Rational expressions are simplified in the following way:

- Factor first.
- Divide out common factors and simplify.
- Restrict the denominator if it contains variables.
(solve for x when denominator is zero)

For example: $\frac{(x+3)}{x}$; $x \neq 0$

Rational expressions are simplified in the following way:

1. Factor first.
2. Divide out common factors and simplify.
3. Restrict the FIRST FACTORED denominator if it contains variables.

Example 1: Simplify the following.

$$\frac{20x^2 - 30x}{5x}$$

Restriction

$$5x \neq 0$$

$$x \neq \frac{0}{5}$$

$$x \neq 0$$

$$= \frac{\cancel{10}^2 x \cancel{(2x-3)}}{\cancel{5x}}$$

$$= 2(2x-3), x \neq 0$$

Example 2

$$\frac{3x-2}{4-6x}$$

$$= \frac{3x-2}{2(2-3x)}$$

$$= \frac{\cancel{(3x-2)}}{-2\cancel{(3x-2)}}$$

Restriction

$$2(2-3x) \neq 0$$

$$2-3x \neq 0$$

$$-3x \neq -2$$

$$x \neq \frac{2}{3}$$

$$= \frac{1}{-2}$$

$$= -\frac{1}{2}, x \neq \frac{2}{3}$$

i.w)

Simplify and state any restrictions on the variables:

①
$$\frac{x^2 - 5x + 6}{x^2 + 3x - 10}$$

Restrictions

$$(x+5)(x-2) \neq 0$$

$$\therefore x \neq -5, 2$$

$$= \frac{(x-2)(x-3)}{(x+5)(x-2)}$$

$$= \frac{x-3}{x+5}, x \neq -5, 2$$

$x+5 \neq 0$
 $x \neq -5$
 $x-2 \neq 0$
 $x \neq 2$

②
$$\frac{x^2 - 7xy + 10y^2}{x^2 + xy - 6y^2}$$

$$= \frac{(x-5y)(x-2y)}{(x+3y)(x-2y)}$$

$$= \frac{x-5y}{x+3y}, x \neq 2y, -3y$$

$x+3y \neq 0$
 $x \neq -3y$
 $x-2y \neq 0$
 $x \neq 2y$

H/w p112 #1-7, 10

1. Solve the following Equations:

(2 marks each)

a) $\frac{2x+1}{3} - \frac{x+1}{4} = 3$

$\times 12 \rightarrow 12 \left(\frac{2x+1}{3} \right) - 12 \left(\frac{x+1}{4} \right) = 12(3)$

$4(2x+1) - 3(x+1) = 36$

$8x + 4 - 3x - 3 = 36$

$5x + 1 = 36$

$5x = 35$

$\therefore x = 7$

b) $16w^2 = 25$

$16w^2 - 25 = 0$

$(4w)^2 - 5^2 = 0$

$(4w+5)(4w-5) = 0$

$4w+5=0$

$w = -\frac{5}{4}$

$4w-5=0$

$w = \frac{5}{4}$

$w^2 = \frac{25}{16}$

$w = \pm \sqrt{\frac{25}{16}}$

$w = \pm \frac{5}{4}$

c) $2y^2 + 5y - 12 = 0$

$2y^2 + 8y - 3y - 12 = 0$

$2y(y+4) - 3(y+4) = 0$

$(2y-3)(y+4) = 0$

$\therefore y = -4, \frac{3}{2}$

2. Evaluate the following:

(2 marks each – show steps!)

a) $\frac{1}{2} - \frac{5}{3} \times 1\frac{2}{5}$

$= \frac{1}{2} - \frac{5}{3} \times \frac{7}{5}$

$= \frac{1}{2} - \frac{35}{15}$

$= \frac{15}{30} - \frac{70}{30}$

$= -\frac{55}{30}$

$= -\frac{11}{6}$

b) $-4\frac{1}{6} \times \left(-7\frac{3}{4}\right)$

$\left(-\frac{25}{6}\right) \times \left(-\frac{31}{4}\right)$

$= \frac{775}{24}$

3. a) Simplify the expression, $5x^2(x-2) - 3(x^3+1)$ *2 marks*

b) Evaluate the above expression for $x = -2$ *1 mark*

4. Factor the following expressions completely:

a. $x^2-11x+24$ *1 mark*

b. $9x^2+24x+12$ *2 marks*

5. For the quadratic relation, $y = -2(x-4)^2 - 5$, find; *1 mark each*

a. The coordinates of the vertex.

b. The equation of the axis of symmetry.

c. The direction of opening.

d. The y-intercept.

$$\underline{E_x} = 2\sqrt{\frac{8}{3}} + \left(\left(\frac{-16}{9} \right) + 0.6 \right) \div 1.1$$

$$= \frac{8}{3} + \left(\frac{-16}{9} + \frac{3}{5} \right) \div \frac{11}{10}$$

$$= \frac{8}{3} + \left(\frac{-80}{45} + \frac{27}{45} \right) \div \frac{11}{10}$$

$$= \frac{8}{3} + \frac{-53}{45} \div \frac{11}{10}$$

$$= \frac{8}{3} + \frac{-53}{45} \times \frac{10^2}{11}$$

$$= \frac{8}{3} + \frac{-530}{495}$$

$$= \frac{8}{3} + \frac{-106}{99}$$

$$= \frac{264}{99} + \frac{-106}{99}$$

$$= \underline{\underline{\frac{158}{99}}}$$

$$0.6 = \frac{6}{10}$$

$$= \frac{3}{5}$$

$$1.1 = \frac{11}{10}$$