2.6 and 2.7 Operations with Rational Expressions

The ability to manipulate rational expressions is an important skill for engineers, scientists, and mathematicians. Some examples of such situations are the calculation of the resistance in parallel circuits and the calculation of the focal length in curved lenses.

When multiplying or dividing rational expressions, follow these steps:

- Factor any polynomials, if possible
- When dividing by a rational expression, multiply by the reciprocal of the rational expression
- Divide by any common factors
- Determine any restrictions

When adding or subtracting rational expressions, follow these steps:

- Factor the denominators
- Determine the least common multiple of the denominators
- Rewrite the expressions with a common denominator
- Add or subtract the numerators
- Simplify and state the restrictions

When considering restrictions, you must include any instance where the denominator can be zero in the original expression.

1. Simplify and state the restrictions on the variable.

a) $$\frac{2x^3}{3x^2-4x} \times \frac{6x-8}{2x-2}$$

$$= \frac{2x^{\cancel{3}\,2}}{\cancel{x}(\cancel{3x-4})} \times \frac{\cancel{2}(\cancel{3x-4})}{\cancel{2}(x-1)}$$

$$= \frac{2x^2}{x-1}, \quad x \neq 0, 1, \frac{4}{3}$$

Restrictions

$x(3x-4) \neq 0$

$\underline{x \neq 0}$ $\quad 3x-4 \neq 0$

$3x \neq 4$

$\underline{x \neq \frac{4}{3}}$

$x-1 \neq 0$

$\underline{x \neq 1}$

c) $$\frac{2x^2-13x+20}{x} \times \frac{x^2+4x-5}{x^2-5x+4}$$

$$= \frac{(2x-5)\cancel{(x-4)}}{x} \times \frac{(x+5)\cancel{(x-1)}}{\cancel{(x-4)}\cancel{(x-1)}}$$

$$\boxed{\begin{aligned} &2x^2-13x+20 \\ &= 2x^2-8x-5x+20 \\ &= 2x(x-4)-5(x-4) \\ &= (2x-5)(x-4) \end{aligned}}$$

$$= \frac{(2x-5)(x+5)}{x}, \quad x \neq 0, 1, 4$$

<u>Note</u> When multiplying expressions of the form $\frac{A}{B} \times \frac{C}{D}$, restriction come from the factored form of B and D (<u>BEFORE</u> CANCELLING!)

2. Simplify and state the restrictions on the variable.

a) $\dfrac{x^2-4}{x+3} \div \dfrac{x^2-x-6}{x^2+x-6}$

$$= \frac{(x-2)(x+2)}{(x+3)_{B}} \div \frac{(x-3)(x+2)^{C}}{(x+3)(x-2)_{D}} \qquad \frac{A}{B} \div \frac{C}{D}$$

$$= \frac{(x-2)\cancel{(x+2)}}{\cancel{(x+3)}} \times \frac{\cancel{(x+3)}(x-2)}{(x-3)\cancel{(x+2)}}$$

$$= \frac{(x-2)(x-2)}{(x-3)}$$

$$= \frac{(x-2)^2}{x-3}, \quad x \neq -3, -2, 2, 3$$

Restrictions

B	C	D
$x+3 \neq 0$	$x-3 \neq 0$	$x+3 \neq 0$
$x \neq -3$	$x \neq 3$	$x \neq -3$
	$x+2 \neq 0$	$x-2 \neq 0$
	$x \neq -2$	$x \neq 2$

Note: When dividing expressions of the form $\frac{A}{B} \div \frac{C}{D}$, the restrictions come from the factored form of B, C and D (BEFORE converting to multiply!)

2.7 Addition and Subtraction Examples

(3) a) $\dfrac{4}{x^2-x-6} + \dfrac{2}{x^2-2x-3}$

$= \dfrac{4}{(x-3)(x+2)} + \dfrac{2}{(x-3)(x+1)}$

$= \dfrac{4(x+1)}{(x-3)(x+2)(x+1)} + \dfrac{2(x+2)}{(x-3)(x+1)(x+2)}$

Note
When adding and subtracting we often need to 'add in' a binomial to the numerator and denominator.

$= \dfrac{4(x+1) + 2(x+2)}{(x-3)(x+2)(x+1)}$

$= \dfrac{4x+4+2x+4}{(x-3)(x+2)(x+1)}$

$= \dfrac{6x+8}{(x-3)(x+2)(x+1)}$

$= \dfrac{2(3x+4)}{(x-3)(x+2)(x+1)}, \quad x \neq -2, -1, 3$

④ b) $\dfrac{3x}{x-7} - \dfrac{x}{7-x}$

$= \dfrac{3x}{x-7} - \dfrac{x(-1)}{(7-x)(-1)}$

$= \dfrac{3x}{x-7} - \dfrac{(-x)}{x-7}$

$= \dfrac{3x-(-x)}{x-7}$

$= \dfrac{4x}{x-7}, \quad x \neq 7$

2.4 Homework

4 b) $\dfrac{-5x^3y^2}{10xy^3}$

$= \dfrac{-5\cancel{x}y^2(x^2)}{-5\cancel{x}y^2(-2y)}$

$= \dfrac{x^2}{-2y}$

$= -\dfrac{x^2}{2y}$, $x, y \neq 0$

Restrictions

$10xy^3 \neq 0$

$xy^3 \neq 0$

$x \neq 0$ $\quad$ $y^3 \neq 0$

$y \neq 0$

c) $\dfrac{2t(5-t)}{5t^2(t-5)}$

$= \dfrac{-2\cancel{t}\cancel{(t-5)}}{5t^{\cancel{2}}\cancel{(t-5)}}$

$= -\dfrac{2}{5t}$, $t \neq 0, 5$

Restrictions

$5t^2(t-5) \neq 0$

$5t^2 \neq 0$ $\quad$ $t-5 \neq 0$

$t \neq 0$ $\quad$ $t \neq 5$

d) $\dfrac{5ab}{15a^4b - 10a^2b^2}$

$= \dfrac{\cancel{5}\cancel{a}\cancel{b}}{\cancel{5}a^{\cancel{2}}\cancel{b}(3a^2-2b)}$

$= \dfrac{1}{a(3a^2-2b)}$, $a, b \neq 0$, $b \neq \dfrac{3a^2}{2}$

Restrictions

$5a^2b(3a^2-2b) \neq 0$

$5a^2b \neq 0$ $\quad$ $3a^2 - 2b \neq 0$

$a^2 \neq 0$ $\quad$ $b \neq 0$ $\quad$ $3a^2 \neq 2b$

$a \neq 0$

or

$b \neq \dfrac{3a^2}{2}$

or

$a \neq \sqrt{\dfrac{2b}{3}}$

Homework 2.6

6 c) $\dfrac{2x^2-x-1}{x^2-x-6} \times \dfrac{6x^2-5x+1}{8x^2+14x+5}$

$2x^2-x-1$
$= 2x^2-2x+x-1$
$= 2x(x-1)+1(x-1)$
$= (2x+1)(x-1)$

$6x^2-5x+1$
$= 6x^2-3x-2x+1$
$= 3x(2x-1)-1(2x-1)$
$= (3x-1)(2x-1)$

$8x^2+14x+5$
$= 8x^2+10x+4x+5$
$= 2x(4x+5)+1(4x+5)$
$= (2x+1)(4x+5)$

$= \dfrac{(\cancel{2x+1})(x-1)}{(x-3)(x+2)} \times \dfrac{(3x-1)(2x-1)}{(\cancel{2x+1})(4x+5)}$

$= \dfrac{(x-1)(3x-1)(2x-1)}{(x-3)(x+2)(4x+5)}, \; x \neq -2, -\frac{5}{4}, -\frac{1}{2}, 3$

7 a) $\dfrac{x^2-5xy+4y^2}{x^2+3xy-28y^2} \times \dfrac{x^2+2xy+y^2}{x^2-y^2}$

$= \dfrac{(\cancel{x-4y})(\cancel{x-y})}{(x+7y)(\cancel{x-4y})} \times \dfrac{(x+y)^{\cancel{2}}}{(\cancel{x+y})(\cancel{x-y})}$

$= \dfrac{x+y}{x+7y}, \; x \neq -7y, -y, y, 4y$

Restrictions

$x+7y \neq 0$
$x \neq -7y$

$x-4y \neq 0$
$x \neq 4y$

$x+y \neq 0$
$x \neq -y$

$x-y \neq 0$
$x \neq y$