3.1(part2) Completing the Square

Warm Up

A. Write a trinomial that is equivalent to:

① $(x-2)^2$
$= x^2 - 4x + 4$

② $(x+4)^2$
$= x^2 + 8x + 16$

③ $(x+\frac{5}{2})^2$
$= x^2 + 5x + \frac{25}{4}$

B. Factor the following:

① $x^2 + 12x + 36$
$= (x+6)^2$

② $x^2 - 8x + 16$
$= (x-4)^2$

③ $x^2 + 3x + \frac{9}{4}$
$= (x+\frac{3}{2})^2$

'Completing the Square' is a method used to rewrite the Standard Form of a quadratic equation into Vertex Form.

($y = ax^2 + bx + c$ to $y = a(x-h)^2 + k$)

eg. Find the vertex of the parabola described by the equation ① $y = x^2 + 12x + 20$

$= (x^2 + 12x + 36 - 36) + 20$

$= [(x+6)^2 - 36] + 20$

$y = (x+6)^2 - 16$

∴ The vertex is at $(-6, -16)$

② $y = -2x^2 + 16x - 4$

$= -2(x^2 - 8x) - 4$

$= -2[x^2 - 8x + 16 - 16] - 4$

$= -2[(x-4)^2 - 16] - 4$

$= -2(x-4)^2 + 32 - 4$

$y = -2(x-4)^2 + 28$

∴ vertex is $(4, 28)$

Algebraic form of 'Completing the Square'

$$y = ax^2 + bx + c$$

$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

$$= a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2 + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 - a\frac{b^2}{4a^2} + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{ab^2}{4a^2} + c$$

$$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a} \quad \text{or} \quad \frac{4ac - b^2}{4a}$$

The vertex is $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$

Solving Equations by Completing the Square

eg. Solve $p^2 + 2p - 63 = 0$

$p^2 + 2p + 1 - 1 - 63 = 0$

$(p+1)^2 - 1 - 63 = 0$

$(p+1)^2 - 64 = 0$

$(p+1)^2 = 64$

$p+1 = \pm\sqrt{64}$

$p+1 = \pm 8$

$p+1 = -8$ $\qquad$ $p+1 = 8$

$\therefore p = -9, 7$

SAMDEB

Practice Page #17

$$7b^2 - 14b - 56 = 0$$

$$7(b^2 - 2b) - 56 = 0$$

$$7(b^2 - 2b + 1 - 1) - 56 = 0$$

$$7[(b-1)^2 - 1] - 56 = 0$$

$$7(b-1)^2 - 7 - 56 = 0$$

$$7(b-1)^2 = 63$$

$$(b-1)^2 = 9$$

$$b - 1 = \pm\sqrt{9}$$

$$\therefore b = 4, -2$$