Unit 3 Quadratic Functions

3.1 Properties of Quadratic Functions

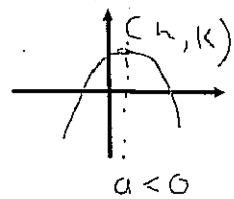
There are 3 common algebraic forms of a quadratic function:

1. Vertex Form

$y = a(x - h)^2 + k$ *(from 'Transformations' in Unit 1, you may prefer to think of this as $y = a(x - d)^2 + c$)*

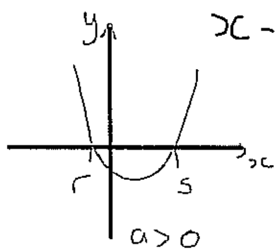
vertex (h, k) (vertex (d, c))

'a' tells us the vertical stretch factor and direction of opening.



2. Factored Form

$y = a(x - r)(x - s)$

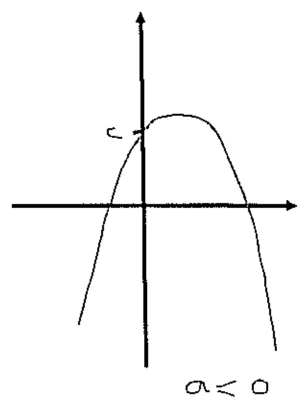


x-intercepts at r and s

3. Standard Form

$y = ax^2 + bx + c$

'c' is the y-intercept



When graphed a quadratic function is parabolic (u-shaped and symmetrical). The vertex of a parabola may be found from the vertex form of the quadratic, (h, k). Alternatively we can find the x- coordinate of the vertex by averaging the zeros, 'r' and 's'.

Second Differences

We can determine whether or not a function is a quadratic from a table of values. A function is quadratic if the second differences are constant. The second difference value is equal to twice the value of 'a' (2a = 2nd diff value)

Examples

1. In each of the following determine whether or not the function is linear or quadratic, then write an equation describing the function:

a)

x	y	1st diff	
-2	5		
-1	1	-4	
0	-3	-4	
1	-7	-4	
2	-11	-4	
3	-15	-4	

$b = -3$

$m = -4$

linear

$y = mx + b$

$\boxed{y = -4x - 3}$

b)

x	y	1st diff	2nd diff
-2	-2		
-1	4	6	
0	6	2	-4
1	4	-2	-4
2	-2	-6	-4
3	-12	-10	-4

$2a = -4$

$a = -2$

vertex $(0, 6)$

quadratic

$y = -2(x - 0)^2 + 6$

$\boxed{y = -2x^2 + 6}$

2. Graph each of the following. State the vertex, direction of opening, y-intercept and equation of the axis of symmetry:

a) $y = -2(x - 4)^2 + 1$

vertex : $(4, 1)$

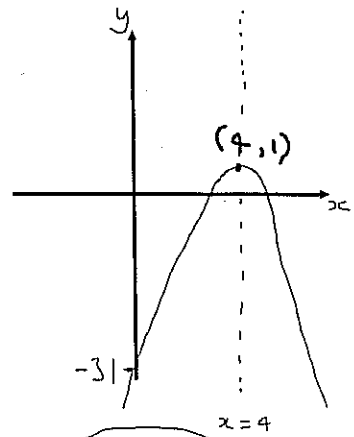
opens down

y-int ($x = 0$) → $y = -2(0-4)^2 + 1$

$= -2(16) + 1$

$\underline{y = -31}$

axis of symmetry : $x = 4$

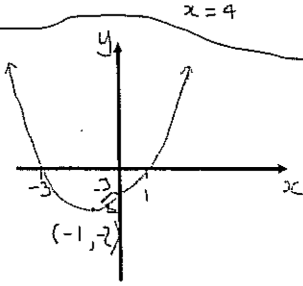


b) $y = 1/2(x - 1)(x + 3)$

zeros at $1, -3$

axis of symmetry : $x = \frac{1 + (-3)}{2}$

$\underline{x = -1}$



y-coordinate of vertex ($x = -1$) → $y = \frac{1}{2}(-1-1)(-1+3)$

$= \frac{1}{2}(-2)(2)$

$= -2$

∴ vertex at $(-1, -2)$

opens up ($a = \frac{1}{2}$)

y-int ($x = 0$) → $y = \frac{1}{2}(0-1)(0+3)$

$= \frac{1}{2}(-1)(3)$

$\underline{y = -\frac{3}{2}}$

c) $y = 2x^2 - 6x + 7$

$y = 2[x^2 - 3x] + 7$

$y = 2[x^2 - 3x + \frac{9}{4} - \frac{9}{4}] + 7$

$= 2[(x - \frac{3}{2})^2 - \frac{9}{4}] + 7$

$= 2(x - \frac{3}{2})^2 - \frac{18}{4} + \frac{28}{4}$

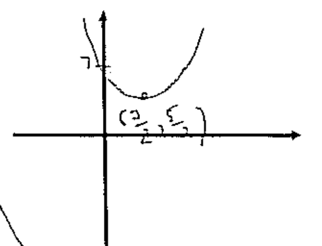
$y = 2(x - \frac{3}{2})^2 + \frac{5}{2}$

∴ vertex is $(\frac{3}{2}, \frac{5}{2})$

opens up

axis of symm : $x = \frac{3}{2}$

y-int : $y = 7$



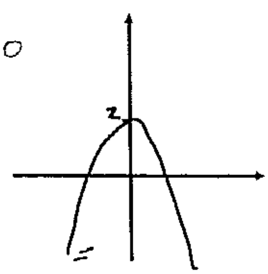
d) $y = -x^2 + 2$

vertex: $(0, 2)$

y-int = 2

opens down

axis of symm : $x = 0$



3. Given the two points (-2, 2) and (4, 2) lie either side of the axis of symmetry of a parabola, determine the equation of the axis of symmetry.

axis of symmetry: $x = \frac{-2+4}{2}$

$\underline{x = 1}$

Homework: p145 #1 - 3, 5-7, 10 - 12