# 3.2 Determining max and min values of Quadratic Functions

In grade 10 we solved problems which involved finding maximum or minimum values.(eg. max height of a rocket) In this chapter you will need to find Maximum or Minimum Profit, Revenue, Demand or Cost:

R(x) is often used to represent a Revenue Function. Revenue is money brought in by selling x items.

p(x) is often used to represent a demand function. Demand is the relationship between price of an item and the number of items sold.

$R(x) = x[p(x)]$

C(x) is often used to represent a Cost Function. C(x) is the total cost of making x items.

P(x)is often used to represent the Profit Function; how much money (profit) is made when x items are sold. Profit = Revenue - Cost

$P(x) = R(x) - C(x)$

$P(x) = x[p(x)] - C(x)$

Read p 151 #3

Finding Max and Min Values Examples

eg. Find the maximum profit of the # of magazines sold that will produce the maximum profit, given $P(x) = -6x^2 + 36x - 48$ where $x$ is the # of magazines sold (in thousands)

in thousands of dollars

Method 1 (Completing the square)

$$P(x) = -6x^2 + 36x - 48$$
$$= -6(x^2 - 6x) - 48$$
$$= -6[x^2 - 6x + 9 - 9] - 48$$
$$= -6[(x-3)^2 - 9] - 48$$
$$= -6(x-3)^2 + 54 - 48$$
$$P(x) = -6(x-3)^2 + 6$$

The max profit occurs at $(3, 6)$
$\therefore$ A max profit of \$6000 is generated when 3000 magazines are sold.

Method 2 (Factoring in order to average the 'zeros')

$$P(x) = -6x^2 + 36x - 48$$
$$= -6(x^2 - 6x + 8)$$
$$P(x) = -6(x-2)(x-4)$$

let $P(x) = 0 \rightarrow \quad 0 = -6(x-2)(x-4)$

$x - 2 = 0 \qquad x - 4 = 0$

$\therefore$ Zeros occur at $x = 2, 4$

x-coordinate of vertex $= \dfrac{2+4}{2}$

$= 3$

y-coordinate of vertex $= P(3)$

$$P(3) = -6(3-2)(3-4)$$
$$= (-6)(1)(-1)$$
$$= 6$$

vertex at $(3, 6)$

(max value of \$6000 when 3000 mags sold)

## Method 3 (Partial Factoring)

$$P(x) = -6x^2 + 36x - 48$$

* common factor the first 2 terms

$$= -6x(x-6) - 48$$

Find $x$ when $P(x) = -48$ → $-48 = -6x(x-6) - 48$

add 48 to both sides → $0 = -6x(x-6)$

$x = 0$     $x - 6 = 0$
$x = 6$

$x$-coordinate of vertex $= \frac{0+6}{2}$

$= \underline{\underline{3}}$

$y$-coordinate of vertex $= P(3)$

$$P(3) = -6(3)^2 + 36(3) - 48$$
$$= -6(9) + 108 - 48$$
$$= -54 + 108 - 48$$
$$= \underline{\underline{6}}$$

vertex at $(3,6)$

(Max of \$6000 occurs at 3000 mags sold)

H/W p153 #1, 2, 4, 5, 7, 8, 11

H/W
#17

$7b^2 - 14b - 56 = 0$

$7(b^2 - 2b) - 56 = 0$

$7(b^2 - 2b + 1 - 1) - 56 = 0$

$7[(b-1)^2 - 1] - 56 = 0$

$7(b-1)^2 - 7 - 56 = 0$

$7(b-1)^2 - 63 = 0$

$7(b-1)^2 = 63$

$(b-1)^2 = 9$

$b - 1 = \pm\sqrt{9}$

$b - 1 = 3 \qquad b - 1 = -3$

$\therefore \underline{\underline{b = 4 \quad , \quad b = -2}}$

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$5v^2 - 21 = 10v$

$5v^2 - 10v - 21 = 0$

$5(v^2 - 2v) - 21 = 0$

$5(v^2 - 2v + 1 - 1) - 21 = 0$

$5[(v-1)^2 - 1] - 21 = 0$

$5(v-1)^2 - 5 - 21 = 0$

$5(v-1)^2 - 26 = 0$

$5(v-1)^2 = 26$

$(v-1)^2 = \frac{26}{5}$

$v - 1 = \pm\sqrt{\frac{26}{5}}$

$v - 1 = \sqrt{\frac{26}{5}} \quad , \quad v - 1 = -\sqrt{\frac{26}{5}}$

$v = 1 + \sqrt{\frac{26}{5}} \quad , \quad v = 1 - \sqrt{\frac{26}{5}}$

$\therefore \underline{\underline{v \doteq 3.28, -1.28}}$