

3.3 Inverse of a Quadratic

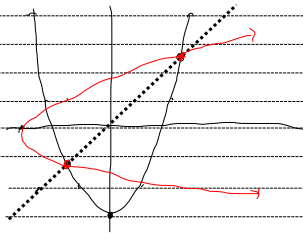
Ex1 Given the fxn $f(x) = x^2 - 3$
Determine the inverse (f^{-1})

$$\begin{array}{ccc} \text{Fxn} & & \text{Inverse (F}^{-1}\text{)} \\ y = x^2 - 3 & \longrightarrow & x = y^2 - 3 \end{array}$$

$$x + 3 = y^2$$

$$y^2 = x + 3$$

$$y = \pm\sqrt{x+3}$$



Ex2 Given $f(x) = 3x^2 + 15x - 10$, find $f^{-1}(-10)$

When given a quadratic in standard form, first rewrite the equation in vertex form by completing the square

$$\begin{aligned} f(x) &= 3(x^2 + 5x) - 10 \\ &= 3\left[x^2 + 5x + \frac{25}{4} - \frac{25}{4}\right] - 10 \\ &= 3\left[\left(x + \frac{5}{2}\right)^2 - \frac{25}{4}\right] - 10 \\ &= 3\left(x + \frac{5}{2}\right)^2 - \frac{75}{4} - \frac{40}{4} \\ y &= 3\left(x + \frac{5}{2}\right)^2 - \frac{115}{4} \end{aligned}$$

$$f^{-1}(x) \rightarrow x = 3\left(y + \frac{5}{2}\right)^2 - \frac{115}{4}$$

$$x + \frac{115}{4} = 3\left(y + \frac{5}{2}\right)^2$$

$$\frac{4x + 115}{4} = 3\left(y + \frac{5}{2}\right)^2$$

$$\frac{4x + 115}{12} = \left(y + \frac{5}{2}\right)^2$$

$$\pm\sqrt{\frac{4x + 115}{12}} = y + \frac{5}{2}$$

$$\therefore f^{-1}(x) = -\frac{5}{2} \pm \sqrt{\frac{4x + 115}{12}}$$

$$f^{-1}(-10) = -\frac{5}{2} \pm \sqrt{\frac{4(-10) + 115}{12}}$$

$$= -\frac{5}{2} \pm \sqrt{\frac{75}{12}}$$

$$= -\frac{5}{2} \pm \sqrt{\frac{25}{4}}$$

$$= -\frac{5}{2} \pm \frac{5}{2}$$

$$\therefore f^{-1}(-10) = 0, -5$$

Homework:

- read p155-160
- complete p161

3-6, 10, 12