3.4 Operations with Radicals

Radicals are expression with square roots.

The square root of any number that is not a perfect square is said to be 'irrational' (cannot be expressed at a fraction)

eg. $\sqrt{2}$ is a radical that is irrational ($\sqrt{2} \doteq 1.4142\ldots$)

$\sqrt{25}$ is a radical that is rational because 25 is a perfect square. ($\sqrt{25} = 5$)

Mixed radicals are radicals with a coefficient

eg. $5\sqrt{3}$ is a mixed radical

Simplifying Radicals (or entire radicals) to Mixed Radicals

Property

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ $(a \geq 0, b \geq 0)$

eg. 1 $\sqrt{75}$

$= \sqrt{(25)(3)}$ — Factor out the greatest perfect square factor

$= \sqrt{25} \times \sqrt{3}$

$= \underline{\underline{5\sqrt{3}}}$

eg 2. $\sqrt{98}$

$= \sqrt{(49)(2)}$

$= \sqrt{49} \times \sqrt{2}$

$= \underline{\underline{7\sqrt{2}}}$

Adding and Subtracting Radicals

eg.1 $11\sqrt{5} + 3\sqrt{5}$

$= \underline{\underline{14\sqrt{5}}}$

eg.2 $13\sqrt{2} - 5\sqrt{2}$

$= \underline{\underline{8\sqrt{2}}}$

eg.3 $\sqrt{12} + 2\sqrt{27}$

$= \sqrt{(4)(3)} + 2\sqrt{(3)(9)}$

$= \sqrt{4}\sqrt{3} + 2\sqrt{3}\sqrt{9}$

$= 2\sqrt{3} + 2\sqrt{3}(3)$

$= 2\sqrt{3} + 6\sqrt{3}$

$= \underline{\underline{8\sqrt{3}}}$

Multiplying and Dividing Radicals

Property

$c\sqrt{a} \times d\sqrt{b} = cd\sqrt{ab}$

$(a \geq 0, b \geq 0)$

eg.1 $(9\sqrt{2})(4\sqrt{7})$

$= \underline{\underline{36\sqrt{14}}}$

eg.2 $(2\sqrt{3})(5\sqrt{6})$

$= 10\sqrt{18}$

$= 10\sqrt{(9)(2)}$

$= 10\sqrt{9}\sqrt{2}$

$= \underline{\underline{30\sqrt{2}}}$

eg.3 $3\sqrt{2}(5 + 2\sqrt{6})$

$= 15\sqrt{2} + 6\sqrt{12}$

$= 15\sqrt{2} + 6\sqrt{(4)(3)}$

$= 15\sqrt{2} + 6\sqrt{4}\sqrt{3}$

$= \underline{\underline{15\sqrt{2} + 12\sqrt{3}}}$

fully simplified ('unlike terms')

eg.4 $(\sqrt{5}+4)(2\sqrt{3}-\sqrt{2})$

$= 2\sqrt{15} - \sqrt{10} + 8\sqrt{3} - 4\sqrt{2}$ ← fully simplified

eg.5 $(\sqrt{6}+3\sqrt{2})(\sqrt{6}-3\sqrt{2})$

$= \sqrt{36} - 3\sqrt{12} + 3\sqrt{12} - 9\sqrt{4}$

$= 6 - 9(2)$

$= 6 - 18$

$= -12$

Homework

read p164-167, complete p167 #1-7, 12

Quiz Wednesday, March 30th

Unit 2 Test

(1) b) $4a^2b^2 \div -2a^2b^3$

$= \frac{-2 \cdot 2 \cdot a \cdot a \cdot b \cdot b}{-2 \cdot a \cdot a \cdot b \cdot b \cdot b}$

$= -\frac{2}{b}$

p153 #5, 7, 11(c)

(5) a) i) $p(x) = -x + 5$

$R(x) = x[p(x)]$

$= x(-x+5)$

$\therefore R(x) = -x^2 + 5x$

ii) $R(x) = -(x^2 - 5x)$

$= -\left(x^2 - 5x + \left(\frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2\right)$

$= -\left[\left(x - \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2\right]$

$= -\left(x - \frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2$

$\therefore R(x) = -\left(x - \frac{5}{2}\right)^2 + \frac{25}{4}$ $\quad \frac{25}{4} \times 1000$

max revenue is \$6 250

(7) c) i) $P(x) = R(x) - C(x)$

$= (-3x^2 + 26x) - (8x + 18)$

$= -3x^2 + 26x - 8x - 18$

$\therefore P(x) = -3x^2 + 18x - 18$

ii) $P(x) = -3(x^2 - 6x) - 18$

$= -3(x^2 - 6x + 9 - 9) - 18$

$= -3[(x-3)^2 - 9] - 18$

$= -3(x-3)^2 + 27 - 18$

$P(x) = -3(x-3)^2 + 9$

$\therefore$ max profit occurs when $x = 3$

(11) c) $P(x) = -5x^2 + 400x - 2550$

solve for x when $P(x) = 4000$

\$4000000 $\div$ 1000

$P(x) = -5(x^2 - 80x) - 2550$

$= -5(x^2 - 80x + 1600 - 1600) - 2550$

$= -5[(x-40)^2 - 1600] - 2550$

$= -5(x-40)^2 + 8000 - 2550$

$P(x) = -5(x-40)^2 + 5450$

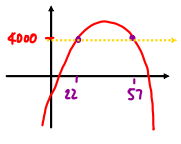
let $P(x) = 4000$ $\rightarrow$ $4000 = -5(x-40)^2 + 5450$

$4000 - 5450 = -5(x-40)^2$

$-1450 = -5(x-40)^2$

$290 = (x-40)^2$

$\pm\sqrt{290} = x - 40$



$x = 40 + \sqrt{290}$, $x = 40 - \sqrt{290}$

$\therefore x \doteq 57.029$, 22.971

Profit of \$4000000 occurs between \$22971 and \$57029