3.5 Solving Quadratic Equations

Warm Up

Expand and Simplify the following expression:

$$[x-(5-\sqrt{2})][x-(5+\sqrt{2})]$$

$$= x^2-(5+\sqrt{2})x-(5-\sqrt{2})x+(5-\sqrt{2})(5+\sqrt{2})$$

$$= x^2-5x-\sqrt{2}x-5x+\sqrt{2}x+25+5\sqrt{2}-5\sqrt{2}-\sqrt{4}$$

$$= x^2-10x+25-2$$

$$= \underline{x^2-10x+23}$$

$$= [(x-5)+\sqrt{2}][(x-5)-\sqrt{2}]$$

$$= (x-5)^2-(\sqrt{2})^2$$

$$= (x-5)^2-2$$

$$= x^2-10x+25-2$$

$$= \underline{x^2-10x+23}$$

Quadratic Formula(proof)

$$ax^2 + bx + c = 0$$

$$a\left(x^2 + \frac{b}{a}x\right) + c = 0$$

$$a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c = 0$$

$$a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2 + c = 0$$

$$a\left(x + \frac{b}{2a}\right)^2 = a\frac{b^2}{4a^2} - c$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - c$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - \frac{4ac}{4a}$$

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Quadratic Formula

Solving by Factoring

eg1 Determine the roots by factoring:

$2x^2 - 7x - 4 = 0$

$2x^2 - 8x + x - 4 = 0$

$2x(x-4) + 1(x-4) = 0$

$(2x+1)(x-4) = 0$

$2x+1=0$ ↙ ↘ $x-4=0$

$2x = -1$ $\quad x = 4$

$x = -\frac{1}{2}$

$\therefore$ The roots are $-\frac{1}{2}$ and 4

Solving using Quadratic Formula

eg 2. Determine the roots by completing the square or using quadratic formula:

$0 = x^2 - 10x + 23$ $\qquad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{10 \pm \sqrt{100 - 4(1)(23)}}{2(1)}$ ← Quadratic formula

$= \frac{10 \pm \sqrt{100-92}}{2}$

$= \frac{10}{2} \pm \frac{\sqrt{8}}{2}$

$= 5 \pm \frac{\sqrt{4}\sqrt{2}}{2}$

$= 5 \pm \sqrt{2}$

$\therefore$ The roots are $5+\sqrt{2}$ and $5-\sqrt{2}$

$x^2 - 10x + 23 = 0$ ← completing the square

$[x^2 - 10x + 25 - 25] + 23 = 0$

$[(x-5)^2 - 25] + 23 = 0$

$(x-5)^2 - 2 = 0$

$(x-5)^2 = 2$

$x - 5 = \pm\sqrt{2}$

$x = 5 \pm \sqrt{2}$

* You can always use technology to check your answer to the solution of a quadratic (observe the x-intercepts)

Homework p177 #1, 2, 3, 4, 7, 8, 9, 10, 11
state the exact value where possible!

Quiz Thursday

- Simplifying Radical Expressions 3.4
- Completing the square (max/min of quadratics) 3.2
- Identifying properties of a quadratic given one of the 3 forms (standard/vertex/factored form) 3.1

H/W Takeup (3.4 p167)

6) b/ $\sqrt{12} + \sqrt{18} - \sqrt{27} + \sqrt{50}$

$= \sqrt{4}\sqrt{3} + \sqrt{9}\sqrt{2} - \sqrt{9}\sqrt{3} + \sqrt{25}\sqrt{2}$

$= 2\sqrt{3} + 3\sqrt{2} - 3\sqrt{3} + 5\sqrt{2}$

$= -\sqrt{3} + 8\sqrt{2}$

$= \underline{8\sqrt{2} - \sqrt{3}}$

7) f/ $(1-\sqrt{3})(2+\sqrt{6})(5+\sqrt{2})$

$= (2 + \sqrt{6} - 2\sqrt{3} - \sqrt{18})(5+\sqrt{2})$

$= (2 + \sqrt{6} - 2\sqrt{3} - \sqrt{9}\sqrt{2})(5+\sqrt{2})$

$= (2 + \sqrt{6} - 2\sqrt{3} - 3\sqrt{2})(5+\sqrt{2})$

$= 10 + 2\sqrt{2} + 5\sqrt{6} + \sqrt{12} - 10\sqrt{3} - 2\sqrt{6} - 15\sqrt{2} - 3\sqrt{4}$

$= 10 + 2\sqrt{2} + 5\sqrt{6} + \sqrt{4}\sqrt{3} - 10\sqrt{3} - 2\sqrt{6} - 15\sqrt{2} - 6$ ($\sqrt{4}\sqrt{3} = 2\sqrt{3}$)

$= 4 - 13\sqrt{2} + 3\sqrt{6} - 8\sqrt{3}$

$= \underline{4 + 3\sqrt{6} - 8\sqrt{3} - 13\sqrt{2}}$