4.2 Working with Exponent Laws

$a^m \times a^n = a^{m+n}$

Product Rule: when multiplying powers with the same base, add exponents. eg. $4^3 \times 4^2 = 4^5$

$a^m \div a^n = a^{m-n}$

Quotient Rule: when dividing powers with the same base, subtract exponents

eg. $4^3 \div 4^2 = 4^1$
$= 4$

$(a^m)^n = a^{m \cdot n}$

Power of a Power Rule: when raising a power to a power, multiply exponents

eg. $(4^3)^2 = 4^6$

$(a \cdot b)^m = a^m \cdot b^m$

Simplifying a power with a product base: the exponent is applied to each part (factor) of the product base.

eg. $(2x)^3 = 2^3x^3$
$= 8x^3$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Simplifying a power with a rational base: the exponent is applied to the numerator and denominator.

eg. $\left(\frac{2}{x}\right)^3 = \frac{2^3}{x^3}$
$= \frac{8}{x^3}$

$a^0 = 1$

if $a \neq 0$, powers with an exponent of zero always equal 1.

eg. $\left(\frac{4a^2c^3}{3x^2y^4}\right)^0 = 1$

$$a^{-m} = \frac{1}{a^m}$$

An integer base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite exponent.

eg. $2^{-3} = \frac{1}{2^3}$

$= \frac{1}{8}$

$$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m} = \frac{b^m}{a^m}$$

The same rule applies to a power with fractional base

eg. $\left(\frac{2}{3}\right)^{-3} = \left(\frac{3}{2}\right)^{3}$

$= \frac{3^3}{2^3}$

$= \underline{\underline{\frac{27}{8}}}$

* When simplifying exponential expressions, BEDMAS rules apply (just like any other algebraic expression)

Example #1

Evaluate each of the following.

(a) $7^3 \times 7^5 \div 7^9$

$= 7^{3+5-9}$

$= 7^{-1}$

$= \frac{1}{7^1}$

$= \frac{1}{7}$

* positive exponents are preferred!

(b) $\left(\frac{2}{3}\right)^{-2}$

$= \left(\frac{3}{2}\right)^2$

$= \frac{3^2}{2^2}$

$= \frac{9}{4}$

(c) $\left(\frac{81}{49}\right)^0$

$= 1$

Example #2

Simplify each of the following.

$a^4 \div a^{-2} = a^6$

$b^5 \div b^6 = b^{-1}$

$c^{-2} \div c^{-2} = c^0$

(a) $(7a^2b^3)(2a^3b)^2$

B
E
D
M
A
S

$= 7a^2b^3(2^2a^6b^2)$

$= 7a^2b^3 4a^6b^2$

$= \underline{28a^8b^5}$

(b) $\frac{\overset{4}{\cancel{20}}a^4b^5c^{-2}}{\cancel{5}a^{-2}b^6c^{-2}}$

$= 4a^6b^{-1}c^0$

$= 4a^6\left(\frac{1}{b}\right)(1)$

$= \frac{4a^6}{b}$

(c) $(2a^3b^{-4})^3$

$= 2^3a^9b^{-12}$

$= 8a^9\left(\frac{1}{b^{12}}\right)$

$= \frac{8a^9}{b^{12}}$

(d) $(3a^4b^{-5})^{-2}$

$= 3^{-2}a^{-8}b^{10}$

$= \left(\frac{1}{3^2}\right)\left(\frac{1}{a^8}\right)b^{10}$

$= \frac{b^{10}}{9a^8}$

p221 #2

f) $[(7^{-3})^{-2}]^{-2}$

$= (7^{6})^{-2}$

$= 7^{-12}$

$= \frac{1}{7^{12}}$ ← expressed as a single power with positive exponent.

11. Evaluate each expression for $x = -2$, $y = 3$, and $n = -1$. Express answers in rational form.

a) $(x^n + y^n)^{-2n}$

c) $\left(\frac{x^n}{y^n}\right)^n$

b) ~~$(x^2)^n(y^{-2n})x^{-n}$~~

d) $\left(\frac{xy^n}{(xy)^{2n}}\right)^{2n}$

a) $(x^n + y^n)^{-2n}$

↖ cannot be simplified using exponent rules

no!

$(3+x)^2 \neq 3^2 + x^2$!

$= x^2+6x+9$

sub $x=-2, y=3, n=-1$:

$((-2)^{-1} + (3)^{-1})^{-2(-1)}$

$= \left(-\frac{1}{2} + \frac{1}{3}\right)^2$

$= \left(-\frac{3}{6} + \frac{2}{6}\right)^2$

$= \left(-\frac{1}{6}\right)^2$

$= \frac{(-1)^2}{6^2}$

$= \frac{1}{36}$

d) $\left(\frac{xy^n}{(xy)^{2n}}\right)^{2n}$

$= \frac{(xy^n)^{2n}}{((xy)^{2n})^{2n}}$

$= \frac{x^{2n}y^{2n^2}}{(x^{2n}y^{2n})^{2n}}$

$= \frac{x^{2n}y^{2n^2}}{x^{4n^2}y^{4n^2}}$

$= x^{2n-4n^2} \cdot y^{2n^2-4n^2}$

$= x^{2n-4n^2}y^{-2n^2}$

$= \frac{x^{2n-4n^2}}{y^{2n^2}}$

sub $x=-2, y=3, n=-1$:

$\frac{(-2)^{2(-1)-4(-1)^2}}{(3)^{2(-1)^2}}$

$= \frac{(-2)^{-2-4(1)}}{3^{2(1)}}$

$= \frac{(-2)^{-6}}{9}$

$= -\frac{1}{2^6}\left(\frac{1}{9}\right)$

$= -\frac{1}{64(9)}$

$= -\frac{1}{576}$

$\boxed{\text{p221 \#1, 4-9, 11, 13, 14, 18}}$