4.3 Rational Exponents and Radicals($\sqrt{\ }$)
(fraction exponents)

A number raised to a rational exponent is equivalent to a radical ($\sqrt{\ }$). The rational exponent, $\frac{1}{n}$, indicates the n^{th} root of the base.

eg. $64^{\frac{1}{3}}$ is equivalent to the third root of 64 (cube root)

$$\boxed{b^{\frac{1}{n}} = \sqrt[n]{b}}$$

, where $b \neq 0$, $n > 1$ and $n \in \mathbb{N}$

Examples Evaluate

a) $25^{\frac{1}{2}} = \sqrt{25}$
$= \underline{\underline{5}}$

b) $8^{\frac{1}{3}} = \sqrt[3]{8}$
$= \underline{\underline{2}}$

c) $81^{\frac{1}{4}} = \sqrt[4]{81}$
$= \underline{\underline{3}}$

d) $100000^{\frac{1}{5}} = \sqrt[5]{100\,000}$
$= \underline{\underline{10}}$

If the numerator of a rational exponent (fraction exponent) is not 1, then

$$\boxed{b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m}$$

, where m and n are positive integers ($m, n \in \mathbb{N}$)

Ex

$125^{\frac{2}{3}}$
$= (\sqrt[3]{125})^2$
$= (5)^2$
$= \underline{\underline{25}}$

Know your basic square, cube and fourth roots:

Base	$\sqrt[2]{\ }$	$\sqrt[3]{\ }$	$\sqrt[4]{\ }$
2	4	8	16
3	9	27	81
4	16	64	256
5	25	125	625

eg1 Evaluate. $625^{\frac{3}{4}}$
$= (\sqrt[4]{625})^3$
$= 5^3$
$= \underline{\underline{125}}$

eg 2. Write in radical form

a) $6^{\frac{4}{3}}$

$= \left(\sqrt[3]{6}\right)^4$ or $\sqrt[3]{6^4}$

b) $3x^{\frac{1}{2}}$

$= \sqrt{3}\,x^{\frac{1}{2}}$

* When writing in radical form, rewrite the coefficients (numbers) using $\sqrt{\ }$, but leave variables in rational exponent form.

ex 3. Write using exponents

a) $\sqrt{34}$

$= 34^{\frac{1}{2}}$

b) $\sqrt[4]{5t^3}$

$= (5t^3)^{\frac{1}{4}}$

$= 5^{\frac{1}{4}} t^{\frac{3}{4}}$

$= \sqrt[4]{5}\, t^{\frac{3}{4}}$ preferred answer

Ex 4. evaluate $16^{0.25}$

$= 16^{\frac{1}{4}}$

$= \sqrt[4]{16}$

$= 2$

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Ex 5.

$\sqrt[4]{\left(\frac{16}{81}\right)^{-1}}$

$= \sqrt[4]{\frac{81}{16}}$

$= \frac{\sqrt[4]{81}}{\sqrt[4]{16}}$

$= \frac{3}{2}$

Ex 6. $(125)^{-\frac{1}{3}}$

$= \left(\frac{1}{125}\right)^{\frac{1}{3}}$

$= \sqrt[3]{\frac{1}{125}}$

$= \frac{\sqrt[3]{1}}{\sqrt[3]{125}}$

$= \frac{1}{5}$

H/W Takeup

(7) f) $3^{-2} - 6^{-2} + \frac{3}{2}(-9)^{-1}$

$= \frac{1}{3^2} - \frac{1}{6^2} + \frac{3}{2}\left(-\frac{1}{9}\right)$

$= \frac{1}{9} - \frac{1}{36} - \frac{3}{18}$

$= \frac{4}{36} - \frac{1}{36} - \frac{6}{36}$

$= -\frac{3}{36}$

$= -\frac{1}{12}$

(18) f) $[(3x^4)^{6-m}]\left(\frac{1}{x}\right)^m$

$= 3^{6-m} x^{4(6-m)} \left(\frac{1}{x}\right)^m$

$= 3^{6-m} \cdot x^{24-4m} \cdot \frac{1}{x^m}$

$= 3^{6-m} \cdot x^{24-4m-m}$

$= 3^{6-m} \cdot x^{24-5m}$

$= 3^{6-m} \cdot x^{24-5m}$ or $3^{6-m} x^{24-5m}$

[redacted]

13 a) $2^3 \times 4^{-2} \div 2^2$

$$= 8 \times \frac{1}{4^2} \div \frac{4}{1}$$

$$= 8 \times \frac{1}{16} \times \frac{1}{4}$$

$$= \frac{8}{64}$$

$$= \frac{1}{8}$$

$$= 2^3 \times (2^2)^{-2} \div 2^2$$

$$= 2^3 \times 2^{-4} \div 2^2$$

$$= 2^{3+(-4)-2}$$

$$= 2^{-3}$$

$$= \frac{1}{2^3}$$

$$= \frac{1}{8}$$