Solving Equations with the same Base

(4.4 continued: not in text)

Examples ① Write each power as a power with base 2:

a) 4^2

$= (2^2)^2$

$= \underline{\underline{2^4}}$

b) 10^0

$= 2^0$ (anything with zero exponent is 1)

c) $\left(\frac{1}{2}\right)^5$

$= (2^{-1})^5$

$= \underline{\underline{2^{-5}}}$

② Solve the following:

a) $2^x = \frac{1}{4}$

$2^x = 4^{-1}$

$2^x = (2^2)^{-1}$

$2^x = 2^{-2}$

$\therefore \underline{\underline{x = -2}}$

b) $2^{x+1} = 64$

$2^{x+1} = 2^6$

$\therefore x+1 = 6$

$\underline{\underline{x = 5}}$

c) $3(5^{2x+1}) = 375$

$\div 3 \rightarrow \quad 5^{2x+1} = 125$

$5^{2x+1} = 5^3$

$\therefore 2x+1 = 3$

$2x = 2$

$\underline{\underline{x = 1}}$

d) $3^{2x} = 9^{3x+1}$

$3^{2x} = (3^2)^{3x+1}$

$3^{2x} = 3^{6x+2}$

$\therefore 2x = 6x+2$

$-4x = 2$

$x = \frac{2}{-4}$

$\underline{\underline{x = -\frac{1}{2}}}$

H/W Complete #1, 2, 4 – 7, 10, 11 in handout

9. Simplify. Express answers in rational form with positive exponents.

a) $(36m^4n^6)^{0.5}(81m^{12}n^8)^{0.25}$

b) $\left(\dfrac{(6x^3)^2(6y^3)}{(9xy)^6}\right)^{-\frac{1}{3}}$

c) $\left(\dfrac{\sqrt{64a^{12}}}{(a^{1.5})^{-6}}\right)^{\frac{2}{3}}$

d) $\left(\dfrac{(x^{18})^{\frac{-1}{6}}}{\sqrt[5]{243x^{10}}}\right)^{0.5}$

$\sqrt[5]{243}\,(x^{10})^{\frac{1}{5}}$

$= 3\,x^{10/5}$

$= \underline{3x^2}$

$= \left(\dfrac{x^{-3}}{3x^2}\right)^{1/2}$

$x^{-3} \div 3x^2$

$\dfrac{x^{-5}}{3}$

$= \dfrac{1}{3x^5}$

$= \left(\dfrac{1}{3x^2}\cdot\left(\dfrac{x}{1}\right)^{-3}\right)^{1/2}$

$= \left(\dfrac{1}{3x^2}\cdot\dfrac{1}{x^3}\right)^{1/2}$

$= \left(\dfrac{1}{3x^5}\right)^{1/2}$

$= \underline{\underline{\dfrac{1}{\sqrt{3}\,x^{5/2}}}}$

1. Find a quadratic algebraic model that best represents your data. Define your variables. Be sure to use data points that are far apart and don't use outlier points. *Hint: You could use the last data entry as the vertex of your parabola.* Use a graphing calculator to check your function's accuracy. Show your work below and write the final algebraic equation of your model **in standard form.** ***(5 marks- application)***

vertex $(96.2, 0.4)$ $P(48.3, 4)$

$$d = a(t-96.2)^2 + 0.4$$

sub $(48.3, 4)$

$$4 = a(48.3-96.2)^2 + 0.4$$

$$4 = a(47.9)^2 + 0.4$$

$$3.6 = a\left(\frac{479}{10}\right)^2$$

$$3.6 = a\left(\frac{479^2}{100}\right)$$

$$\frac{360}{479^2} = a$$

$$d = \frac{360}{479^2}(t-96.2)^2 + 0.4$$

$$d = \frac{360}{479^2}\left(t^2 - 192.4t + \left(\frac{962}{10}\right)^2\right) + 0.4$$

$$d = \frac{360}{479^2}\left(t^2 - \frac{1924}{10}t + \frac{962^2}{100}\right) + \frac{4}{10}$$

4. Find the inverse of this function, state its domain and range, and describe in words what the inverse of the function models. ***(4 marks - application)***

$$d = \frac{360}{479^2}(t-96.2)^2 + 0.4$$

$$d - 0.4 = \frac{360}{479^2}(t-96.2)^2$$

$$\frac{(d-0.4)479^2}{360} = (t-96.2)^2$$

$$t - 96.2 = \pm\sqrt{\frac{(d-0.4)479^2}{360}}$$

$$t = \pm\sqrt{\frac{(d-0.4)479^2}{360}} + 96.2$$

5. Use the inverse of the function to find the time that the water was 3 cm above the hole. ***(1 mark - application)***

$$t = \pm\sqrt{\frac{(3-0.4)479^2}{360}} + 96.2$$

$= \_\_\_\_\_\_$ seconds

6. If you were given a container with the same diameter and hole size as the one in your experiment but unlimited height: Using your algebraic model, how high should you fill the water level so that the container would drain empty at the one hour mark? (show your work) ***(3 marks - application)***

$$d = \frac{360}{479^2}(t-96.2)^2 + 0.4$$

↓ need vertex at $(3600, 0.4)$

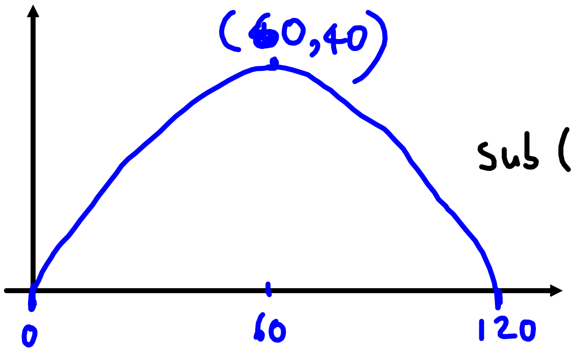
$$d = \frac{360}{479^2}(t-3600)^2 + 0.4$$

sub $t=0$

7. How would you expect the function to change if you began with the same water level as before but doubled the diameter of the hole? Give an example of what the new model might look like in vertex form. ***(2 marks - application)***

2. A ping pong ball, after being projected upward off the ground, reached a height of 40 cm and it travelled a horizontal distance of 120 cm. Use this data to determine the equation of the parabolic path of the ball, **in either factored or vertex form**. Please include a labeled diagram as part of your solution. *Hint: Use one end of the path as the origin (at (0, 0)).*

(4 marks)



$y = a(x-60)^2 + 40$

sub $(0,0) \rightarrow \quad 0 = a(0-60)^2 + 40$

$0 = 3600a + 40$

$a = -\frac{40}{3600}$

$a = -\frac{1}{90}$

$\therefore\ y = -\frac{1}{90}(x-60)^2 + 40$

or $\quad y = a(x-120)(x-0)$

$y = ax(x-120)$

sub $(60,40) \rightarrow 40 = a(60)(60-120)$

$40 = -3600a$

$a = -\frac{1}{90}$

$\therefore\ y = -\frac{1}{90}x(x-120)$

3. $3x^2 - 2x - 1 + k = 0$

one real root means $b^2 - 4ac = 0$

$$(-2)^2 - 4(3)(-1+k) = 0$$

$$4 - 12(-1+k) = 0$$

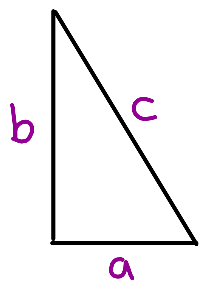
$$4 + 12 - 12k = 0$$

$$-12k = -16$$

$$k = \frac{-16}{-12}$$

$$k = \frac{4}{3}$$

4. The difference between the length of the hypotenuse(the longest side) and the length of the next longest side of a right triangle is 3 cm. The difference between the length of the two perpendicular('short') sides is 3 cm. Find the length of the longest side. *(include a labelled diagram or 'let statement' in your answer). Hint: $a^2 + b^2 = c^2$* ***(4 marks)***



b

$c = b + 3$

$a = b - 3$

c

$b = c - 3$

$a = c - 6$

$$a^2 + b^2 = c^2$$

$$(c-6)^2 + (c-3)^2 = c^2$$

$$c^2 - 12c + 36 + c^2 - 6c + 9 = c^2$$

$$2c^2 - 18c + 45 = c^2$$

$$c^2 - 18c + 45 = 0$$

$$(c - 15)(c - 3) = 0$$

$$\therefore c = 15, \cancel{3}$$

inadmissable

$\therefore$ The longest side is 15cm