4.4 Simplifying and Evaluating Exponential expressions - Warm Up

1. Evaluate.

a) $(27)^{\frac{2}{3}}$

b) $(125)^{-\frac{1}{3}}$

c) $\left(\frac{1}{16}\right)^{-\frac{3}{4}}$

d) $\left(\frac{32}{243}\right)^{-\frac{2}{5}}$

H/W p235 #5, 7, 9

① a) $27^{\frac{2}{3}} = \sqrt[3]{(27)^2}$ or $(\sqrt[3]{27})^2$

$= (\sqrt[3]{27})^2$

$= 3^2$

$= 9$

b) $125^{-\frac{1}{3}}$

$= \left(\frac{1}{125}\right)^{\frac{1}{3}}$

$= \frac{1}{\sqrt[3]{125}}$

$= \frac{1}{5}$

c) $\left(\frac{1}{16}\right)^{-\frac{3}{4}}$

$= (16)^{\frac{3}{4}}$

$= (\sqrt[4]{16})^3$

$= 2^3$

$= 8$

(2) a) $x^{\frac{3}{5}} \times x^{\frac{2}{5}} = x^{\frac{5}{5}}$

$= x$

b) $n^{\frac{1}{2}} \times n^{\frac{1}{3}} \times n^{\frac{1}{4}}$

$= n^{\frac{6}{12}} \times n^{\frac{4}{12}} \times n^{\frac{3}{12}}$

$= n^{\frac{13}{12}}$

c) $(9x^2y^4)^{\frac{1}{2}} \times (125x^6y^3)^{\frac{2}{3}}$

$= (9^{\frac{1}{2}}x^1y^2)(125^{\frac{2}{3}}x^4y^2)$

$= 3x^1y^2(\sqrt[3]{125})^2x^4y^2$

$= 3x^1y^2 25x^4y^2$

$= 75x^5y^4$

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d) $(27y^3)^{\frac{1}{3}} \times \left(\frac{1}{16y^4}\right)^{-\frac{3}{4}}$

$= \sqrt[3]{27}\, y^1 (16y^4)^{\frac{3}{4}}$

$= 3y(\sqrt[4]{16})^3 y^3$

$= 3y\,8y^3$

$= 24y^4$

(3) a) $(100xy)^{\frac{1}{2}}(25x^3y^2)^{-\frac{1}{2}}$

$= 10x^{\frac{1}{2}}y^{\frac{1}{2}}\left(\frac{1}{25x^3y^2}\right)^{\frac{1}{2}}$

$= 10x^{\frac{1}{2}}y^{\frac{1}{2}}\left(\frac{1}{5x^{\frac{3}{2}}y^1}\right)$

$= \frac{10x^{\frac{1}{2}}y^{\frac{1}{2}}}{5x^{\frac{3}{2}}y^1}$

$= 2x^{-\frac{2}{2}}y^{-\frac{1}{2}}$

$= 2x^{-1}y^{-\frac{1}{2}}$

$= 2 \cdot \frac{1}{x} \cdot \frac{1}{y^{\frac{1}{2}}}$

$= \frac{2}{x^1y^{\frac{1}{2}}}$

b) $(27x^6)^{\frac{2}{3}} \div (9x^4)^{\frac{1}{2}}$

$= (\sqrt[3]{27})^2x^4 \div 3x^2$

$= 9x^4 \div 3x^2$

$= \underline{3x^2}$

c) $(64x^2y^4)^{\frac{1}{2}} \div (16x^2y^4)^{\frac{1}{4}}$

$= 8x^1y^2 \div 2x^{\frac{1}{2}}y^1$

$= 4x^{\frac{1}{2}}y^1$ or $\underline{4\sqrt{x}y}$

d) $\frac{x^{-\frac{2}{3}}}{x^{-\frac{4}{5}}}$

$= \frac{x^{-\frac{10}{15}}}{x^{-\frac{12}{15}}}$

$= \underline{x^{\frac{2}{15}}}$

14. State whether each expression is true or false.

a) $9^{\frac{1}{2}} + 4^{\frac{1}{2}} = (9 + 4)^{\frac{1}{2}}$

b) $9^{\frac{1}{2}} + 4^{\frac{1}{2}} = (9 \times 4)^{\frac{1}{2}}$

c) $\left(\frac{1}{a} + \frac{1}{b}\right)^{-1} = a + b$

d) $\left(\frac{1}{a} \times \frac{1}{b}\right)^{-1} = ab$

e) $\left(x^{\frac{1}{3}} + y^{\frac{1}{3}}\right)^{6} = x^2 + y^2$

f) $\left[\left(x^{\frac{1}{3}}\right)\left(y^{\frac{1}{3}}\right)\right]^{6} = x^2y^2$

e) LS $(x^{\frac{1}{3}}+y^{\frac{1}{3}})^6$

$(x^{\frac{1}{3}}+y^{\frac{1}{3}})(x^{\frac{1}{3}}+y^{\frac{1}{3}})(\ \ldots\ )(\ \ldots\ )\ldots$

results in 2^6 terms (64 terms) which simplify down to 7 unlike terms

(learned in unit 7 : Binomial expansion)

∴ false

f) $\left[(x^{\frac{1}{3}})(y^{\frac{1}{3}})\right]^6$

$= (x^{\frac{1}{3}})^6 (y^{\frac{1}{3}})^6$ $\quad (ab)^n = a^n b^n$

$= x^{\frac{6}{3}} y^{\frac{6}{3}}$

$= x^2 y^2$

∴ True

c) $\left(\frac{1}{a} + \frac{1}{b}\right)^{-1}$

$= \left(\frac{b}{ab} + \frac{a}{ab}\right)^{-1}$

$= \left(\frac{a+b}{ab}\right)^{-1}$

$= \frac{ab}{a+b}$

$\neq a+b$

∴ false

6. Write as a single power, then evaluate. Express answers in rational form.

a) $4^{\frac{1}{5}}(4^{0.3})$

b) $100^{0.2}(100^{\frac{-7}{10}})$

c) $\frac{64^{\frac{4}{3}}}{64}$

d) $\frac{27^{-1}}{27^{\frac{-2}{3}}}$

e) $\frac{(16^{-2.5})^{-0.2}}{16^{\frac{3}{4}}}$

f) $\frac{(8^{-2})(8^{2.5})}{(8^6)^{-0.25}}$

e) $\frac{(16^{-2.5})^{-0.2}}{16^{\frac{3}{4}}}$

$= \frac{(16^{-\frac{5}{2}})^{-\frac{1}{5}}}{16^{\frac{3}{4}}}$

$= \frac{16^{\frac{5}{10}}}{16^{\frac{3}{4}}}$

$= \frac{16^{\frac{1}{2}}}{16^{\frac{3}{4}}}$

$= 16^{\frac{1}{2}-\frac{3}{4}}$

$= 16^{\frac{2}{4}-\frac{3}{4}}$

$= 16^{-\frac{1}{4}}$

$= \frac{1}{16^{\frac{1}{4}}}$

$= \frac{1}{\sqrt[4]{16}}$

$= \frac{1}{2}$

18. Solve.

a) $\left(\frac{1}{16}\right)^{\frac{1}{4}} - \sqrt[3]{\frac{8}{27}} = \sqrt{x^2}$

b) $\sqrt[3]{\frac{1}{8}} - \sqrt[4]{x^4} + 15 = \sqrt[4]{16}$

$\frac{\sqrt[3]{1}}{\sqrt[3]{8}} - (x^4)^{\frac{1}{4}} + 15 = 2$

$\frac{1}{2} - x^{\frac{4}{4}} + 15 = 2$

$\frac{1}{2} - x + \frac{30}{2} = \frac{4}{2}$

$\times 2 \rightarrow \quad 1 - 2x + 30 = 4$

$-2x = 4 - 30 - 1$

$-2x = -25$

$x = 12.5$