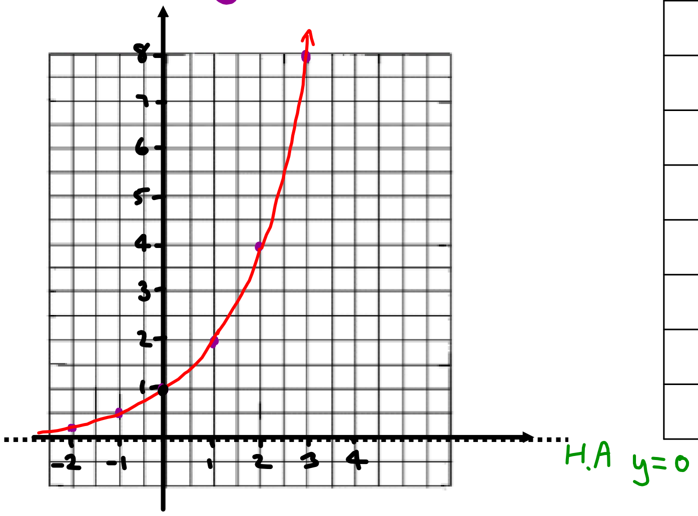
4.5 Properties of Exponential functions

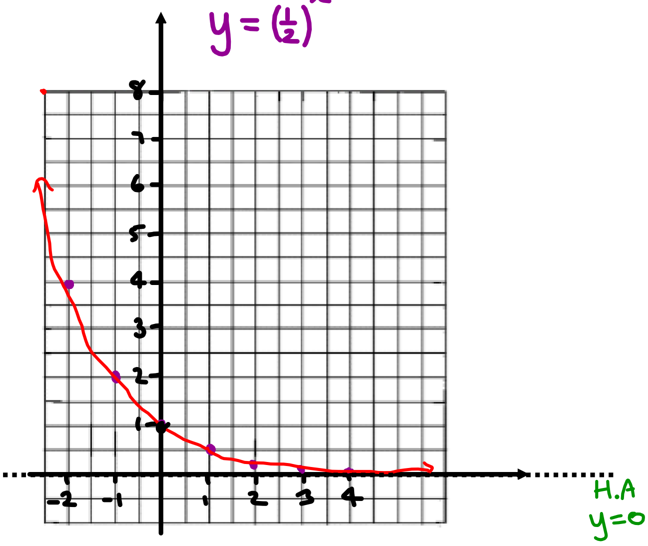
$y = 2^x$



x	$y=2^x$
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
4	$2^4 = 16$

This graph demonstrates 'Exponential Growth'

$y = (\frac{1}{2})^x$



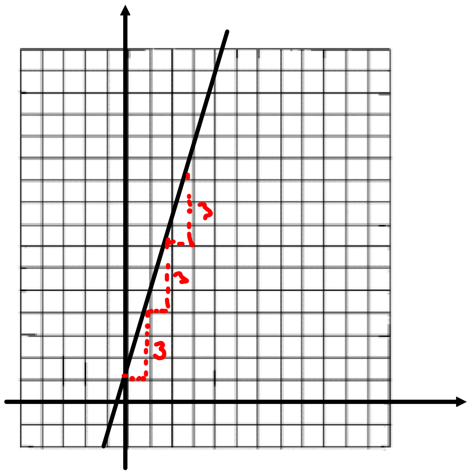
x	$y=(\frac{1}{2})^x$
-2	$(\frac{1}{2})^{-2} = 4$
-1	$(\frac{1}{2})^{-1} = 2$
0	$(\frac{1}{2})^0 = 1$
1	$(\frac{1}{2})^1 = \frac{1}{2}$
2	$(\frac{1}{2})^2 = \frac{1}{4}$
3	$(\frac{1}{2})^3 = \frac{1}{8}$
4	$(\frac{1}{4}) = \frac{1}{16}$

This graph demonstrates 'Exponential Decay'

Give a function of the form $y = b^x$, exponential growth occurs when $b > 1$ (eg. $y = 1.5^x$ is always increasing as x increases ie. exponential growth)
Exponential decay occurs when $0 < b < 1$ (eg. $y = (\frac{1}{4})^x$ is always decreasing as x increases ie. exponential decay)

Using first and second differences:

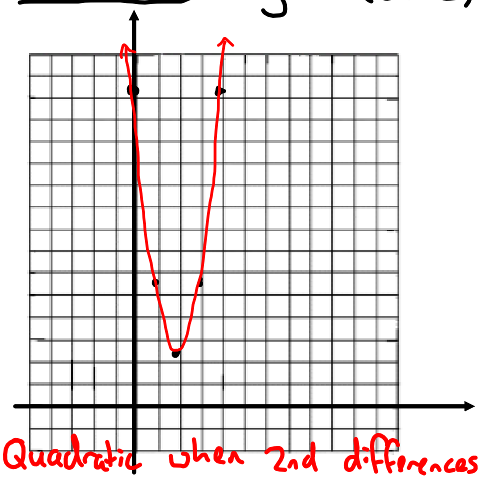
Linear $y = 3x + 1$



x	y	First Diff
-2	-5	
-1	-2	$-2-(-5) = 3$
0	1	$1-(-2) = 3$
1	4	$4-1 = 3$
2	7	$7-4 = 3$
3	10	$10-7 = 3$
4	13	$13-10 = 3$

Linear when First Differences are equal.

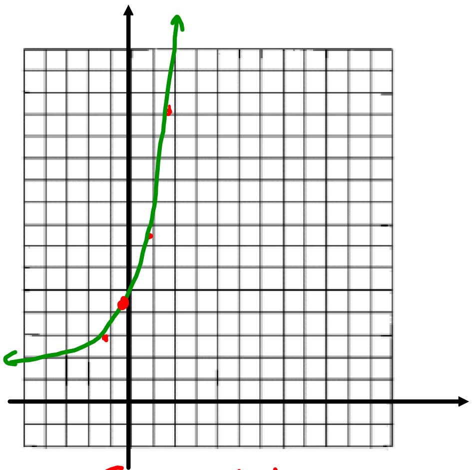
Quadratic $y = 3(x-2)^2 + 2$



x	y	1st Diff	2nd Diff
-2	50		
-1	29	-21	
0	14	-15	6
1	5	-9	6
2	2	-3	6
3	5	3	6
4	14	9	6

Quadratic when 2nd differences are equal

Exponential equations of the form $y = ab^x$ $y = 3(2^x)$



x	y	1st Diff	2nd Diff
-1	$\frac{3}{2}$		
0	3	$\frac{3}{2}$	
1	6	3	$\frac{3}{2}$
2	12	6	3
3	24	12	6
4	48	24	12
5	96	48	24

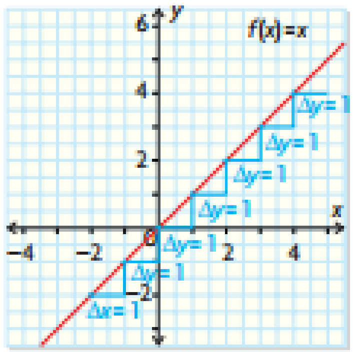
Exponential when first differences increase or decrease by the same factor.

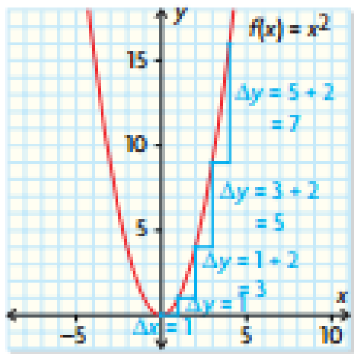
In Summary

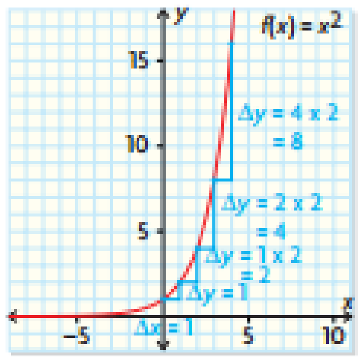
Key Ideas

- Linear, quadratic, and exponential functions have unique first-difference patterns that allow them to be recognized.

Linear	Quadratic	Exponential
Linear functions have constant first differences.	Quadratic functions have first differences that are related by addition. Their second differences are constant.	Exponential functions have first differences that are related by multiplication. Their second finite differences are not constant.







- The exponential function $f(x) = b^x$ is
  - an increasing function representing growth when $b > 1$
  - a decreasing function representing decay when $0 < b < 1$

Need to Know

- The exponential function $f(x) = b^x$ has the following characteristics:
  - If $b > 0$, then the function is defined, its domain is $\{x \in \mathbb{R}\}$, and its range is $\{y \in \mathbb{R} \mid y > 0\}$.
  - If $b > 1$, then the greater the value of b, the faster the growth.
  - If $0 < b < 1$, then the lesser the value of b, the faster the decay.
  - The function has the x-axis, $y = 0$, as horizontal asymptote.
  - The function has a y-intercept of 1.
- Linear, quadratic, and exponential functions can be recognized from their graphs. Linear functions are represented by straight lines, quadratic functions by parabolas, and exponential functions by quickly increasing or decreasing curves with a horizontal asymptote.
- A function in which the variables have exponent 1 (e.g., $f(x) = 2x$) is linear. A function with a single squared term (e.g., $f(x) = 3x^2 - 1$) is quadratic. A function with a positive base (0 and 1 excluded) and variable exponent (e.g., $f(x) = 5^x$) is exponential.

Sketching the Parent/Base Exponential Function:

Key Points for $y=b^x$

$(-1, b^{-1})$ or $(-1, \frac{1}{b})$

$(0, b^0)$ or $(0, 1)$ H.A. $y=0$

$(1, b^1)$ or $(1, b)$

eg. When sketching $y=3^x$, the key points would be: $(-1, \frac{1}{3})$

$(0, 1)$ H.A $y=0$

$(1, 3)$

eg. Sketch $y = 2\left(\frac{1}{2}\right)^{x+1} - 4$

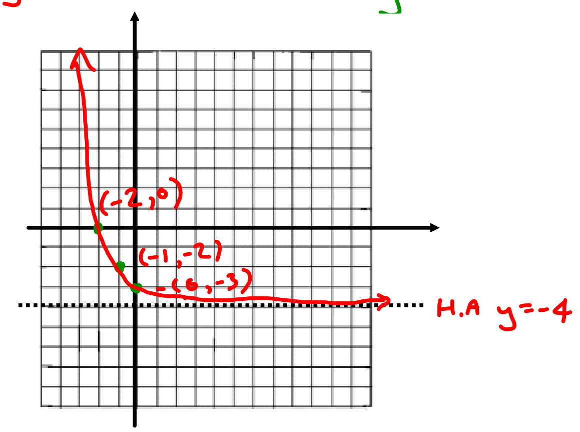
$a=2$, $d=-1$, $c=-4$

$g(x) = af[k(x-d)]+c$

$g(x) = a(b^{k(x-d)}) + c$

$y=\left(\frac{1}{2}\right)^x$ Key Points		$y=2\left(\frac{1}{2}\right)^{x+1}-4$
$(-1, 2)$	$\longrightarrow$	$(-2, 0)$
$(0, 1)$	$\longrightarrow$	$(-1, -2)$
$(1, \frac{1}{2})$	$\longrightarrow$	$(0, -3)$
H.A. $y=0$	$\longrightarrow$	H.A. $y=-4$

	x	y
R	—	—
S	—	$a=2$ 2
T	$d=-1$ -1	$c=-4$ -4



Homework: p243# 1 and 2

graph $y = -2(3^{x-4})$ using 'Desmos', then by hand using an RST chart

1. Solve.

a) $2^x = 16$
b) $3^x = 27$
c) $2^x = 128$
d) $5^x = 125$
e) $4^y = 256$
f) $729 = 9^z$
g) $(-3)^x = -27$
h) $(-2)^x = -32$
i) $(-5)^n = 25$
j) $81 = (-3)^x$
k) $-2^x = -16$
l) $-4^y = -64$
m) $-5^x = -625$
n) $(-1)^x = 1$
o) $(-1)^m = -1$

2. Solve.

a) $7^{w-2} = 49$
b) $3^{x+4} = 27$
c) $2^{1-x} = 128$
d) $4^{3k} = 64$
e) $5^{3x-1} = 25$
f) $-81 = -3^{2x+6}$
g) $4^{x-1} = 1$
h) $3^{2-2x} = 1$
i) $(-1)^{2x} = 1$

3. Solve and check.

a) $6^{x+3} = 6^{2x}$
b) $2^{x+3} = 2^{2x-1}$
c) $3^{2y+3} = 3^{y+5}$
d) $2^{4x-7} = 2^{2x+1}$

7. Solve and check.

a) $4^x = 8$
b) $64^z = 16$
c) $(-8)^y = -2$
d) $9^{-x} = 3$
e) $2^{9x} = \frac{1}{8}$
f) $9^{6x} = \frac{1}{27}$
g) $2^x = 16^4$
h) $2^{-2g} = 32$
i) $9^{2y+1} = 27$

8. Solve and check.

a) $9^{x+1} = 27^{2x}$
b) $16^y = 64^{2y-1}$
c) $36^{t-2} = 216^{-2t}$
d) $8^{2x-1} = 16^{x-1}$
e) $25^{1-3x} = 125^{-x}$
f) $16^{3+k} = 32^{1-2k}$

9. Solve and check.

a) $5 = 25^{\frac{x}{2}}$
b) $8 = 2^{\frac{x}{3}}$
c) $9^{\frac{y}{5}} = 27$
d) $\frac{1}{2} = 2^{\frac{a}{3}}$
e) $4^{\frac{x}{4}} = \frac{1}{8}$
f) $\left(\frac{3}{2}\right)^{\frac{m}{2}} = \frac{4}{9}$

e) $7^{5d-1} = 7^{2d+5}$

f) $3^{b-5} = 3^{2b-3}$

4. Solve.

a) $16^{2x} = 8^{3x}$

b) $4^{t} = 8^{t+1}$

c) $27^{x-1} = 9^{2x}$

d) $25^{2-r} = 125^{2r-4}$

e) $16^{2p+1} = 8^{3p+1}$

f) $(-8)^{1-2x} = (-32)^{1-x}$

5. Solve and check.

a) $2^{x+5} = 4^{x+2}$

b) $2^{x} = 4^{x-1}$

c) $9^{2q-6} = 3^{q+6}$

d) $4^{x} = 8^{x+1}$

e) $27^{y-1} = 9^{2y-4}$

f) $8^{x+3} = 16^{2x+1}$

6. Solve and check.

a) $5^{4-x} = \frac{1}{5}$

b) $10^{y-2} = \frac{1}{10\,000}$

c) $6^{3x-7} = \frac{1}{6}$

d) $3^{3x-1} = \frac{1}{81}$

e) $5^{2n+1} = \frac{1}{125}$

f) $\frac{1}{256} = 2^{2-5w}$

10. Solve.

a) $3(5^{x+1}) = 15$

b) $2(3^{y-2}) = 18$

c) $5(4^{x}) = 10$

d) $2(4^{v+1}) = 1$

e) $2 = 6(3^{4f-2})$

f) $27(3^{3x+1}) = 3$