4.7 Applications Involving Exponential Functions

Warm Up:

1. The volume of water in a container increases by 8% every day. If the initial volume is 100ml, what will the volume be in 3 days? $b = 1.08$

$$V = 100(1.08)^3$$
$$\doteq \underline{125.97\text{ ml}}$$

Exponential Growth

2. The value of a vehicle depreciates by 20% every year. If the initial value of the vehicle is $20000, what will the value be in 5 years? $b = 0.8$

$$\text{Value} = 20000(0.8)^5$$
$$\doteq \underline{\$6553.60}$$

Exponential Decay

Given an equation of the form $y = ab^x$, Exponential Growth occurs when $b > 1$, Exponential Decay occurs when $0 < b < 1$. 'a' represents the initial value.

Ex 2. Complete the table.

	Function	Exponential Growth or Decay?	Initial Value	Growth or Decay Rate
a)	$V(t) = 20(1.02)^t$	growth	20	2%
b)	$P(n) = (0.8)^n$	decay	1	20%
c)	$A(x) = 0.5(3)^x$	growth	0.5	200%
d)	$Q(w) = 600\left(\frac{5}{8}\right)^w$	decay	600	$\frac{3}{8}$ or 37.5%

Key Ideas

- The exponential function $f(x) = ab^x$ and its graph can be used as a model to solve problems involving exponential growth and decay. Note that
  - $f(x)$ is the final amount or number
  - a is the initial amount or number
  - for exponential growth, $b = 1 +$ growth rate; for exponential decay, $b = 1 -$ decay rate
  - x is the number of growth or decay periods

Need to Know

- For situations that can be modeled by an exponential function:
  - If the *growth rate* (as a percent) is given, then the base of the power in the equation can be obtained by *adding* the rate, as a decimal, to 1. For example, a growth rate of 8% involves multiplying repeatedly by 1.08.
  - If the *decay rate* (as a percent) is given, then the base of the power in the equation is obtained by *subtracting* the rate, as a decimal, from 1. For example, a decay rate of 8% involves multiplying repeatedly by 0.92.
  - One way to tell the difference between growth and decay is to consider whether the quantity in question (e.g., light intensity, population, dollar value) has increased or decreased.
  - The units for the growth/decay rate and for the number of growth/decay periods must be the same. For example, if light intensity decreases "per metre," then the number of decay periods in the equation is measured in metres, too.

Day1

H/W p261 #1–7

#~~2~~

In application problems, we often see functions of the form $f(x) = ab^x$. Sometimes we need to solve 'time' problems like:

- the time it takes an amount to double...
- the half-life of a sample is ....

In these problems, we model the final Amount ($f(t)$) against time (t): $f(t) = ab^{\frac{t}{d}}$, where t is the time variable and d is the duration of each half-life or doubling period.

eg. A 200 g sample of radioactive polonium-210 has a *half-life* of 138 days. This means that every 138 days, the amount of polonium left in a sample is half of the original amount. The mass of polonium, in grams, that remains after t days can be modelled by $M(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{138}}$.

a) Determine the mass that remains after 5 years.
b) How long does it take for this 200 g sample to decay to 110 g?

initial value

duration of the half-life (d)

base (b)

a) Consistent units of time measure needed!

$$5 \text{ years} = (5 \times 365) \text{ days} = 1825 \text{ days}$$

$$M(1825) = 200\left(\tfrac{1}{2}\right)^{\frac{1825}{138}}$$

$$\doteq 0.02089 \text{ grams}$$

b) sub $M(t) = 110g$

$$110 = 200\left(\tfrac{1}{2}\right)^{\frac{t}{138}}$$

$$\frac{110}{200} = \left(\tfrac{1}{2}\right)^{\frac{t}{138}}$$

$$0.55 = \left(\tfrac{1}{2}\right)^{\frac{t}{138}}$$

S
A
M
D
E
B

grade 11

guess and check

let $x = \frac{t}{138}$

$$0.55 = \left(\tfrac{1}{2}\right)^x$$

$\left(\frac{1}{2}\right)^1 = 0.5$

$\left(\frac{1}{2}\right)^2 = 0.25$

$\left(\frac{1}{2}\right)^{0.9} = 0.5358$

$\left(\frac{1}{2}\right)^{0.85} = 0.5548$

$\left(\frac{1}{2}\right)^{0.86} = 0.551$

$\therefore x \doteq 0.86$

$0.86 = \frac{t}{138}$

$t \doteq 118.68$

$\therefore$ Approx 119 days to reach 110 grams

grade 12

logarithms

$$\log 0.55 = \log\left(\tfrac{1}{2}\right)^{\frac{t}{138}}$$

$$\log 0.55 = \frac{t}{138}\log\left(\tfrac{1}{2}\right)$$

$$\frac{\log 0.55}{\log\left(\frac{1}{2}\right)} = \frac{t}{138}$$

$$t = \frac{138 \log 0.55}{\log 0.5}$$

$\therefore t \doteq 119.02$

Classwork (day 2) p261 #9-13

When given an equation of the form $y = ab^x$, 'a' represents the y-intercept.

When given an equation of the form $y = ab^x + c$, 'a+c' represents the y-intercept.

Unit 4 Review Homework

- p267 #4, 5, 8, 9, 11, 13, 14, 15, 17
- Unit Test on Tuesday, April 26th