5.1 Trigonometric Ratios of Acute Angles

In this lesson, three trigonometric ratios (secant, cosecant and cotangent) will be defined and applied. These involve ratios of the lengths of the sides in a right triangle.

In a right triangle, one angle is 90º and the side across from this angle is called the hypotenuse. The two sides which form the 90º angle are called the legs of the right triangle. We show a right triangle below. The legs are defined as either "opposite" or "adjacent" (next to) the angle A.

We shall call the opposite side "opp," the adjacent side "adj" and the hypotenuse "hyp."

$$\frac{\sin A}{\cos A} = \frac{opp}{hyp} \div \frac{adj}{hyp}$$

$$= \frac{opp}{\cancel{hyp}} \times \frac{\cancel{hyp}}{adj}$$

$$= \frac{opp}{adj}$$

$$= \tan A$$

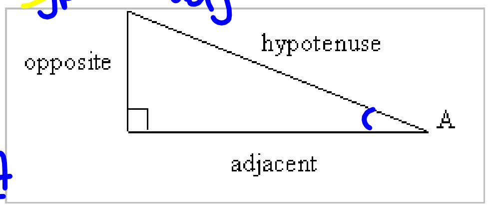
Definitions:
(SOHCAHTOA)

i. $\sin(A) = \frac{opp}{hyp}$

ii. $\cos(A) = \frac{adj}{hyp}$

iii. $\tan(A) = \frac{opp}{adj} = \frac{\sin(A)}{\cos(A)}$

The Primary Trig. Ratios



Three more trigonometric ratios can be defined as the reciprocals of these fundamental ratios.

Reciprocals are two expressions that have a product of 1 (e.g., 4 and $\frac{1}{4}$ or x and $\frac{1}{x}$).

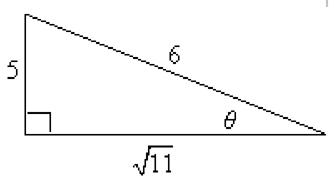
They are **cosecant, secant, and cotangent**. The trigonometric reciprocals are mainly useful for simplifying integration and trigonometric identities.

The ratios are given by the following equations:

iv. **csc(A)** = $\frac{1}{\sin(A)} = \frac{hyp}{opp}$

v. **sec(A)** = $\frac{1}{\cos(A)} = \frac{hyp}{adj}$

vi. **cot(A)** = $\frac{1}{\tan(A)} = \frac{adj}{opp}$



For example, in the triangle shown beside, with respect to angle θ, opp = 5, hyp = 6, and by the Pythagorean Theorem, adj = $\sqrt{11}$.

By the three definitions we have:

csc(θ) = $\frac{hyp}{opp} = \frac{6}{5}$	sec(θ) = $\frac{hyp}{adj} = \frac{6}{\sqrt{11}}$	cot(θ) = $\frac{adj}{opp} = \frac{\sqrt{11}}{5}$

Often, fractions involving radicals are rewritten so that there is not a radical expression in the denominator.

Rationalizing the denominator we would get sec(θ) = $\frac{6}{\sqrt{11}} = \frac{6\sqrt{11}}{11}$.

We can use any of these three reciprocal trig ratios to find the measure of an unknown angle θ.

For example, if sec(θ) = 1.7, we can use a calculator to determine what angle has this secant.

Using a TI-83 calculator, we press **2nd** **COS** [(1.7) **x^{-1}**] and get $\theta \approx 53.97°$. We use the **x^{-1}** key because cosine is the reciprocal of secant and we need to invert the value 1.7 and use the **COS** key since the calculator does not have a secant key.

eg. Rationalize the denominators below:

a) $\frac{1}{\sqrt{2}}$ $\frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}}$ $= \frac{\sqrt{2}}{2}$

b) $\frac{2}{\sqrt{13}}$ $= \frac{\sqrt{13}}{\sqrt{13}} \times \frac{2}{\sqrt{13}}$ $= \frac{2\sqrt{13}}{\sqrt{169}}$ $= \frac{2\sqrt{13}}{13}$

1. Determine the measure of each angle, to the nearest degree, if the angles are in the first quadrant.

a) cot A = 7

$\frac{1}{\tan A} = \frac{7}{1}$

$\tan A = \frac{1}{7}$ ← reciprocate both sides

$A = \tan^{-1}\left(\frac{1}{7}\right)$

$\therefore A \doteq 8.1°$

b) sec B = $\frac{7}{3}$

c) csc C = $\frac{11}{8}$

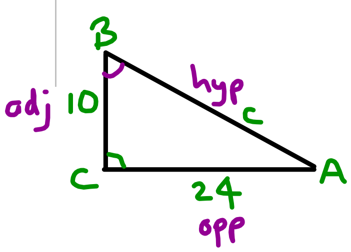
$\frac{1}{\sin C} = \frac{11}{8}$

$\sin C = \frac{8}{11}$

$C = \sin^{-1}\left(\frac{8}{11}\right)$

$\therefore C \doteq 46.7°$

2. In △ABC, if $a = 10$, $b = 24$, and ∠C = 90°, determine the exact expressions for the six trigonometric ratios for ∠B.



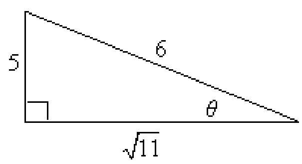
$c = \sqrt{10^2 + 24^2}$

$c = 26$

$\sin B = \frac{24}{26} = \frac{12}{13}$, $\cos B = \frac{10}{26} = \frac{5}{13}$

$\tan B = \frac{24}{10} = \frac{12}{5}$, $\csc B = \frac{13}{12}$, $\sec B = \frac{13}{5}$, $\cot B = \frac{5}{12}$

3. In the triangle below, find angle θ using **a)** one of the primary trigonometric ratios and **b)** one of the reciprocal trig ratios.



a) $\tan\theta = \frac{5}{\sqrt{11}}$

$\theta = \tan^{-1}\left(\frac{5}{\sqrt{11}}\right)$

$\theta \doteq 56.4°$

b) $\sec\theta = \frac{6}{\sqrt{11}}$

$\cos\theta = \frac{\sqrt{11}}{6}$

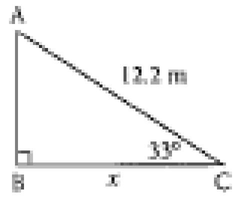
$\theta = \cos^{-1}\left(\frac{\sqrt{11}}{6}\right)$

$\theta \doteq 56.4$

Homework: Page 280 #1 - 8, 11

SOHCAHTOA Applications

1. Find the length of BC to the nearest tenth of a metre.

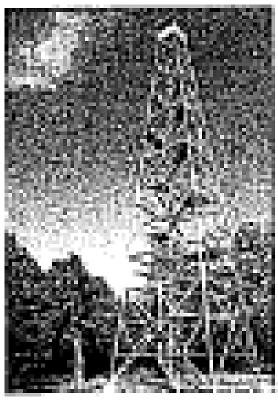


2. 

From a point 4.5 m from the base of a wind turbine, the angle of elevation to the top of the turbine is 87°. Find the height of the wind turbine to the nearest tenth of a metre.

3. From a point 9.3 m from the base of a billboard, the angle of elevation to the top of the billboard is 28°. Find the height of the billboard to the nearest tenth of a metre.

4. A forest ranger is in a fire tower 120 ft above the ground. She sights a fire at an angle of depression of 3°. How far is the fire from the base of the tower, to the nearest foot?



5. From the top of a 38.5 m-high cliff, the angle of depression to a boat is 38°. How far is the boat from the base of the cliff, to the nearest metre?

6. A 4-m long ladder is leaning up against the side of a garage. It reaches 3.8 m up the side of the garage wall. Find the angle the ladder makes with the ground, to the nearest degree.

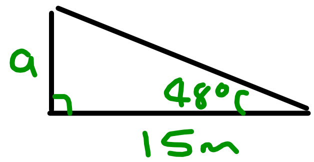
7. A lighthouse sits at the top of a cliff. The top of the lighthouse is 41 m above the water. The angle of depression to a small sailboat is 22°. Describe how you would find the distance from the sailboat to the base of the cliff.

8. Tonya is standing 17 m from the base of a tower. She measures the angle of elevation to the top of a tower to be 33°. What is the height of the tower, to the nearest metre?

9. A flagpole casts a shadow 22 m long when the sun's rays make an angle of 30° with the ground. How tall is the flagpole, to the nearest metre?

10. Marlene is making a pen in her backyard for her daughter's pet rabbits. She makes the pen in the shape of a right triangle. Two sides of the pen each measure 3 m. What is the length of the third side?

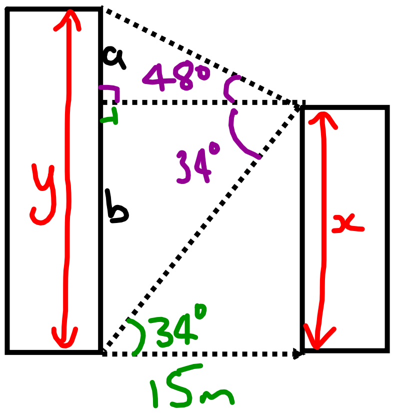
11. Two buildings are 15 m apart. From the top of the shorter building, the angle of elevation to the top of the taller building is 48°, and the angle of depression to the bottom of the taller building is 34°. Find the heights of the two buildings to the nearest tenth of a metre.

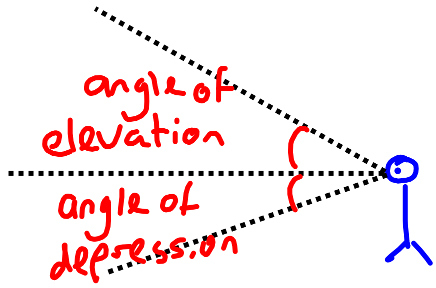


$\tan 48° = \frac{a}{15}$

$a = 15 \tan 48°$

$\doteq \underline{16.7\text{ m}}$





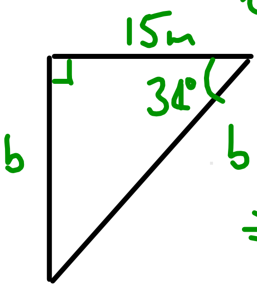
$y = a + b$

$\doteq 16.7 + 10.1$

$\doteq \underline{\underline{26.8\text{ m}}}$

$x = b$

$= \underline{\underline{10.1\text{ m}}}$



$\tan 34° = \frac{b}{15}$

$b = 15 \tan 34°$

$\doteq \underline{10.1\text{ m}}$

Answers

1. 10.2 m
2. 85.9 m
3. 4.9 m
4. 2290 ft
5. 49 m
6. 72°
7. 101 m
8. 11 m
9. 13 m
10. 4.2 m
11. 10.1 m and 26.8 m