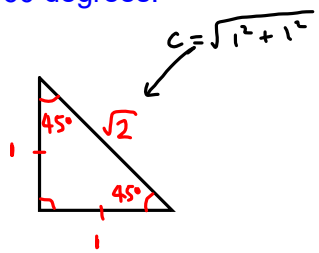
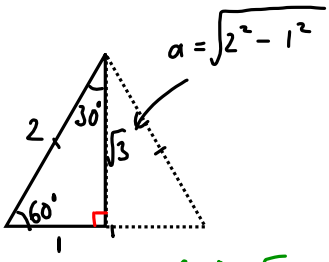
5.2 Special Triangles

Special Triangles are used to determine the exact values of side lengths in triangles that have angles of 30, 45, or 60 degrees:





$\sin 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$

$\cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45° = \frac{1}{1} = 1$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

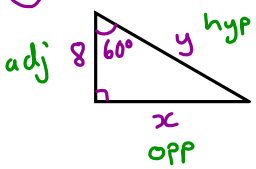
$\tan 30° = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

* always rationalize denominators

Examples: Find the values of x and y in each triangle:

①



$\tan\theta = \frac{opp}{adj}$

$\tan 60° = \frac{x}{8}$

$8\tan 60° = x$

$8\sqrt{3} = x$

$y = \sqrt{(8\sqrt{3})^2 + 8^2}$

$= \sqrt{64(3)+64}$

$= \sqrt{256}$

$\therefore y = 16$

or

$\cos 60° = \frac{8}{y}$

$y\cos 60° = 8$

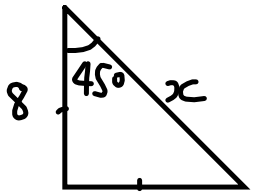
$y = \frac{8}{\cos 60°}$

$= \frac{8}{\frac{1}{2}}$

$= 8 \times \frac{2}{1}$

$y = 16$

(2)



$$\cos 45° = \frac{8}{x}$$

$$x \cos 45° = 8$$

$$x = \frac{8}{\cos 45°}$$

$$= \frac{8}{\sqrt{2}/2}$$

$$= 8 \times \frac{2}{\sqrt{2}}$$

$$= \frac{16}{\sqrt{2}}$$

$$= \frac{16\sqrt{2}}{2}$$

$$= \underline{8\sqrt{2}}$$

Sketch the figure that is described. Find the requested measure.

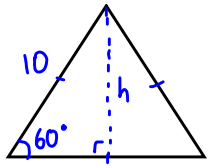
3. An equilateral triangle has a side length of 10 inches. Find the length of the triangles altitude.

4. The altitude of an equilateral triangle is 18 inches. Find the length of a side.

~~5. The perimeter of a square is 48 meters. Find the length of a diagonal.~~

perpendicular height from base to top of triangle

3.



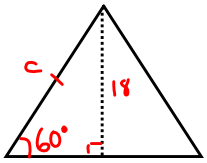
$\sin 60° = \frac{h}{10}$

$10 \sin 60° = h$

$10\left(\frac{\sqrt{3}}{2}\right) = h$

$\therefore h = 5\sqrt{3}$

4.



$\sin 60° = \frac{18}{c}$

$c \sin 60° = 18$

$c = \frac{18}{\sin 60°}$

$= \frac{18}{\sqrt{3}/2}$

$= 18\left(\frac{2}{\sqrt{3}}\right)$

$= \frac{36}{\sqrt{3}}$

$= \frac{36\sqrt{3}}{3}$

$= 12\sqrt{3}$

$\csc 60° = \frac{c}{18}$

$\frac{2}{\sqrt{3}} = \frac{c}{18}$

$\frac{36}{\sqrt{3}} = c$

$c = \frac{36\sqrt{3}}{3}$

$= 12\sqrt{3}$

5.

10. Determine the exact value of each trigonometric expression.

a) $\tan(45) = 1$

b) $\cos(30)\sin(45) = \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} = \frac{\sqrt{3}-\sqrt{2}}{2}$

c) $(\sin 45°)(\cos 45°) + (\sin 30°)(\sin 60°) = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{4}}{4} + \frac{\sqrt{3}}{4}$

$= \frac{2}{4} + \frac{\sqrt{3}}{4}$

$= \frac{2+\sqrt{3}}{4}$

d) $(1 - \tan 45°)(\sin 30°) - 2(\sin 45°) = (1-1)\left(\frac{1}{2}\right) - 2\left(\frac{\sqrt{2}}{2}\right)$

$= (0)\left(\frac{1}{2}\right) - \sqrt{2}$

$= -\sqrt{2}$

H/W p286 #1-9, 11

$\tan^2 60°$ means $(\tan 60°)^2$