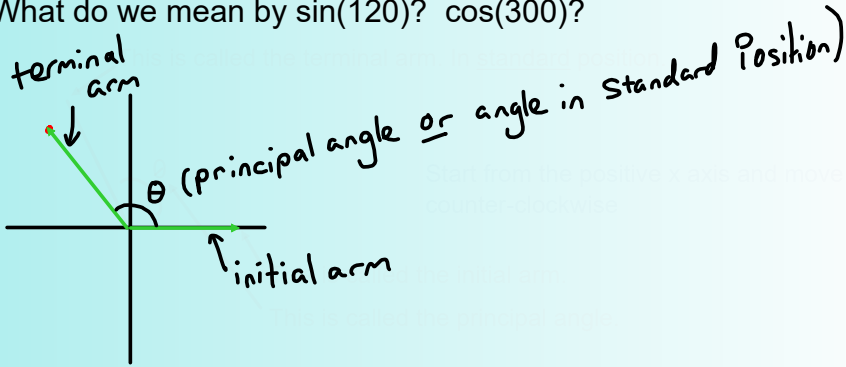


5.3 Trig. angles greater than 90°

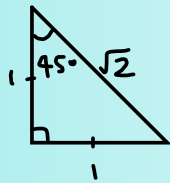
What do we mean by $\sin(120)$? $\cos(300)$?



Standard Position is an angle in the Cartesian plane whose vertex lies at the origin and whose **initial arm** lies on the positive x-axis. The **Principal Angle** (θ) is measured from the **initial arm** to the **terminal arm**. (counterclockwise)

A point on a Cartesian Plane is often used to describe an angle in Standard Position.

Recall our Special Triangles:

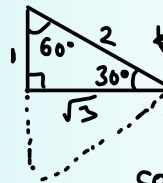


$$\cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = 1$$

$$\sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\cos \theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{4}} = \frac{\sqrt{2}}{2}$$



$$\cos 60^\circ = \frac{1}{2}$$

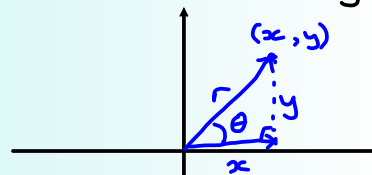
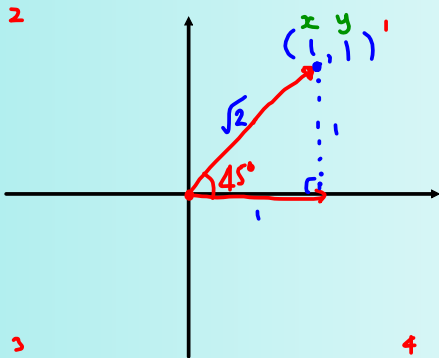
$$\tan 60^\circ = \sqrt{3}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 30^\circ = \frac{1}{2}$$

$$\tan 30^\circ = \frac{\sqrt{3}}{3}$$

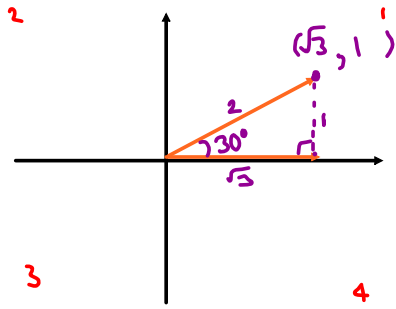


$\sin \theta = \frac{y}{r}$
$\cos \theta = \frac{x}{r}$
$\tan \theta = \frac{y}{x}$

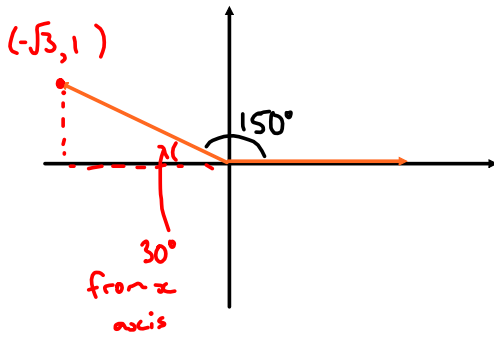
$$\sin 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{1}{1} = 1$$



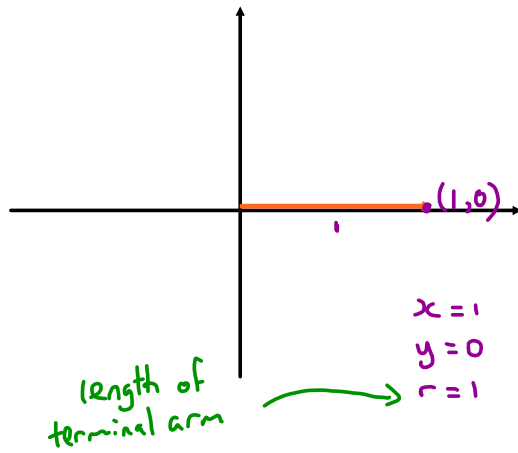
$$\begin{aligned} \sin 30^\circ &= \frac{1}{2} \left(\frac{y}{r} \right) \\ \cos 30^\circ &= \frac{\sqrt{3}}{2} \left(\frac{x}{r} \right) \\ \tan 30^\circ &= \frac{1}{\sqrt{3}} \left(\frac{y}{x} \right) \\ &= \frac{\sqrt{3}}{3} \end{aligned}$$



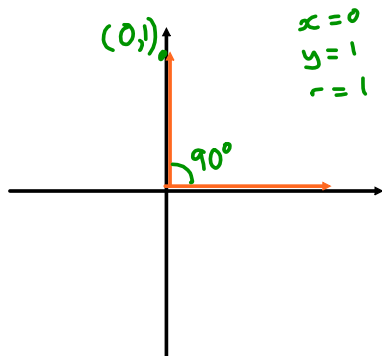
$$\begin{aligned} \sin 150^\circ &= \frac{1}{2} \\ &= \frac{1}{2} \\ \cos 150^\circ &= -\frac{\sqrt{3}}{2} \\ &= -\frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{(-\sqrt{3})^2 + 1^2} \\ &= \sqrt{3 + 1} \\ &= \underline{\underline{2}} \end{aligned}$$

$$\begin{aligned} \tan 150^\circ &= \frac{1}{-\sqrt{3}} \\ &= -\frac{\sqrt{3}}{3} \end{aligned}$$



$$\begin{aligned} \sin 0^\circ &= \frac{0}{1} \\ &= 0 \\ \cos 0^\circ &= \frac{1}{1} \\ &= 1 \\ \tan 0^\circ &= \frac{0}{1} \\ &= 0 \end{aligned}$$

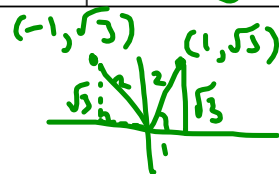


$$\begin{aligned} \sin 90^\circ &= \frac{1}{1} \\ &= 1 \\ \cos 90^\circ &= \frac{0}{1} \\ &= 0 \\ \tan 90^\circ &= \frac{1}{0} \\ &= \underline{\underline{\text{undefined}}} \end{aligned}$$

Homework - complete the chart

MCR3U1 Exact Values of Trigonometric Ratios

θ	$\sin \theta \left(\frac{y}{r}\right)$	$\cos \theta \left(\frac{x}{r}\right)$	$\tan \theta \left(\frac{y}{x}\right)$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
Q1 45°	$\frac{\sqrt{2}}{2}$ +	$\frac{\sqrt{2}}{2}$ +	1 +
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
... 90°	1	0	undefined
120°	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
Q2 135°	$\frac{\sqrt{2}}{2}$ +	$-\frac{\sqrt{2}}{2}$ -	-1 -
150°	$\frac{1}{2}$ +	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
-- 180°	0	-1	0
210°	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
Q3 225°	$-\frac{\sqrt{2}}{2}$ -	$-\frac{\sqrt{2}}{2}$ -	1 +
240°	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$
-- 270°	-1	0	undefined
300°	$-\frac{\sqrt{3}}{2}$ -	$\frac{1}{2}$	$-\sqrt{3}$
Q4 315°	$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$ +	-1 -
330°	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$
360°	0	1	0



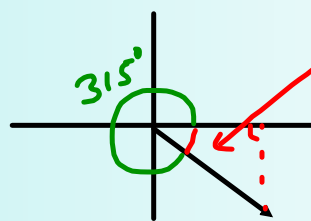
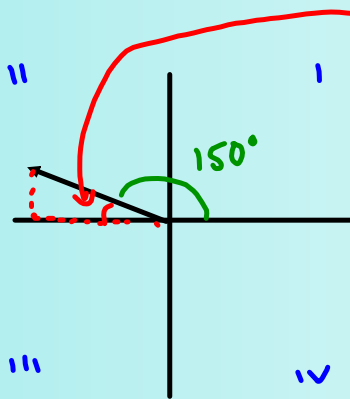
$$\frac{1}{-2} = \frac{-1}{2} = \underline{\underline{-\frac{1}{2}}}$$

Sketch lines at 150° , 315°

Find the **related acute angles**.

The related acute angle of 150° is 30° ($180^\circ - 150^\circ$)

The related acute angle is the angle between the terminal arm and the x-axis.



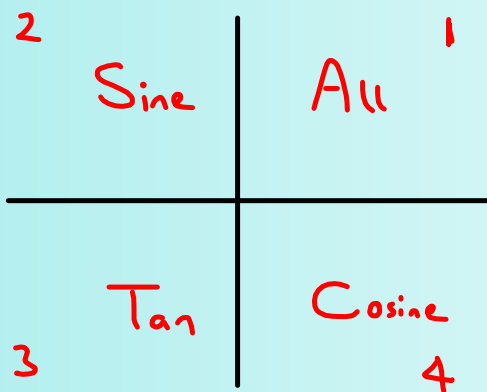
related acute angle = 45°
($360^\circ - 315^\circ$)

Note: a negative angle is an angle measured clockwise from the positive x-axis.

i.e. The terminal arms of a 150° angle and a -210° angle are in the same position on the coordinate grid, and so their primary trig. ratio values would be equal.

(co-terminal)

There is a trick to finding the sign of a trig ratio greater than 90°



Label the quadrants using CAST!

Quadrant 1 - All positive

Quadrant 2 - Only cosine positive

Quadrant 3 - Only sine positive

Quadrant 4 - Only tan positive

Trigonometric Ratio	Quadrant			
	1	2	3	4
sine	+	+	-	-
cosine	+	-	-	+
tangent	+	-	+	-

Complete the worksheet

Angles (Degrees)	Quadrant	Related acute angle	Other angles with same related acute angle (ignoring sign)	Sine Ratio	Cosine Ratio	Tangent Ratio
45						
170						
230						
-80						
					$= -\frac{\sqrt{2}}{2}$	-1
2 options				$= -\frac{\sqrt{3}}{2}$		
270						





