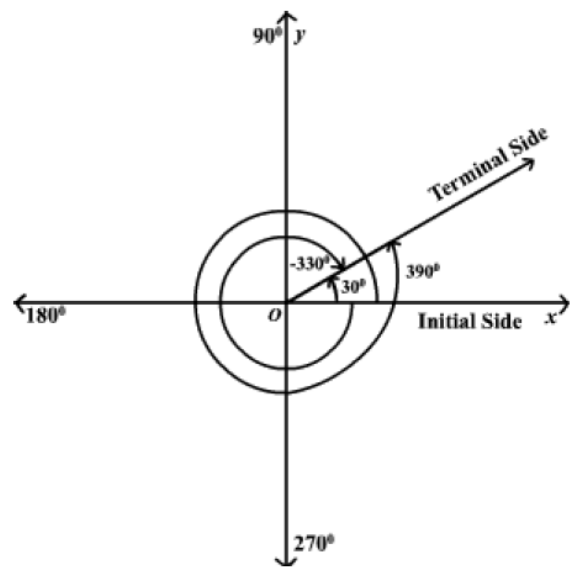


5.4 – Evaluating Trig Ratios for any Angle

When solving problems involving trigonometric quantities, there is almost always more than one solution – sometimes an infinite number of solutions. It is important to find all possible solutions and then select which solutions are appropriate for the problem.

In this section, you will learn how to identify different angles that have the same trigonometric ratio, as well as learn how they are related.

Co-terminal angles – angles in standard position that have the same terminal arm



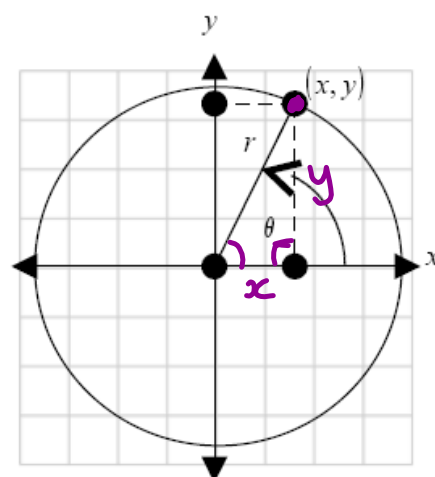
$-330^\circ, 30^\circ, 390^\circ$
are co-terminal

Key points you need to know:

- The primary trigonometric ratios for the angle θ in standard position that has a point (x, y) on its terminal arm can be calculated as

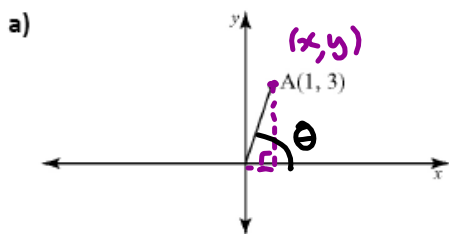
$$\sin \theta = \frac{y}{r}, \cos \theta = \frac{x}{r}, \tan \theta = \frac{y}{x}, \text{ where } r = \sqrt{x^2 + y^2}$$

- For any given sine ratio, two distinct angles between 0° and 360° have this sine ratio
- For any given cosine ratio, two distinct angles between 0° and 360° have this cosine ratio
- For any given tangent ratio, two distinct angles between 0° and 360° have this tangent ratio
- Pairs of related angles can be found using the coordinates of the endpoints of their terminal arms. Use a reference angle in the first quadrant.
- Co-terminal angles are angles with the same terminal arm. They can be positive or negative.



A positive angle is generated when the terminal arm moves in a counter clockwise direction. A negative angle is generated when the terminal arm moves in a clockwise direction.

1. For each point given on the terminal arm of the angle, determine the exact primary values for the trigonometric ratios of the angle.



$$r = \sqrt{1^2 + 3^2}$$

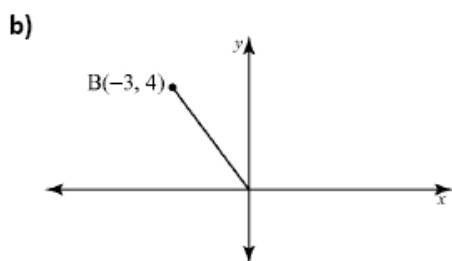
$$= \sqrt{10}$$

$$\cos \theta = \frac{1}{\sqrt{10}}$$

$$= \frac{\sqrt{10}}{10}$$

$$\sin \theta = \frac{3}{\sqrt{10}}$$

$$= \frac{3\sqrt{10}}{\sqrt{10}\sqrt{10}}$$

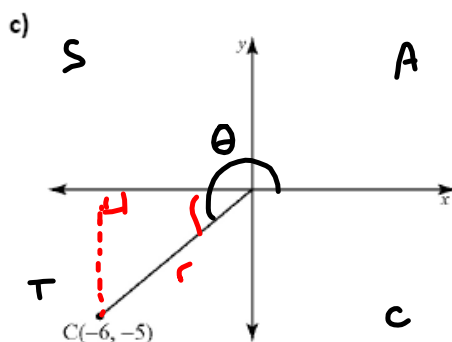


$$\tan \theta = \frac{3}{-1}$$

$$= \underline{\underline{-3}}$$

$$= \frac{3\sqrt{10}}{\sqrt{100}}$$

$$= \frac{3\sqrt{10}}{10}$$



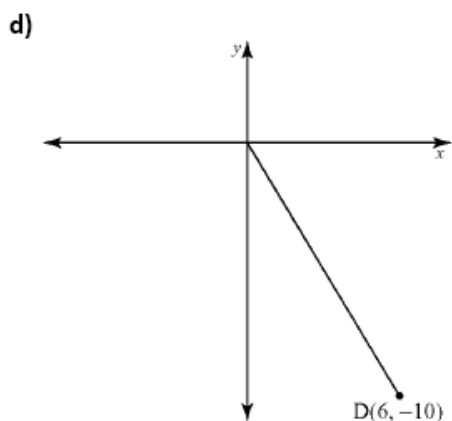
$$r = \sqrt{(-6)^2 + (-5)^2}$$

$$= \sqrt{61}$$

$$\sin \theta = \frac{-5}{\sqrt{61}}$$

$$\cos \theta = \frac{-6}{\sqrt{61}}$$

$$= \underline{\underline{\frac{-6\sqrt{61}}{61}}}$$



$$= \underline{\underline{\frac{-5\sqrt{61}}{61}}}$$

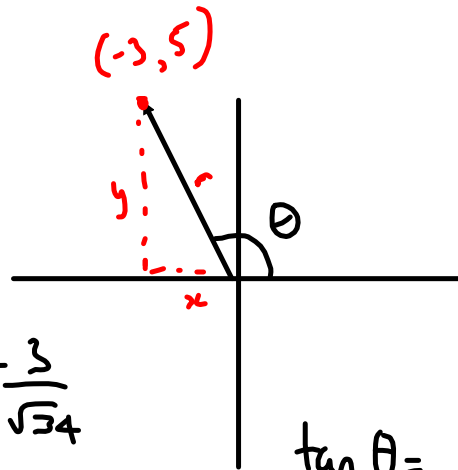
$$\tan \theta = \frac{-5}{-6}$$

$$= \underline{\underline{\frac{5}{6}}}$$

2. The coordinates of a point on the terminal arm of an angle are given. Determine exact expressions for the primary trigonometric ratios for these angles.

- a) E(2, 3) b) F(-3, 5)
c) G(-2, -7) d) H(7, -4)

$$\begin{aligned} \text{b) } r &= \sqrt{(-3)^2 + 5^2} \\ &= \sqrt{34} \end{aligned}$$



$$\begin{aligned} \sin \theta &= \frac{5}{\sqrt{34}} \\ &= \frac{5\sqrt{34}}{34} \end{aligned}$$

$$\begin{aligned} \cos \theta &= \frac{-3}{\sqrt{34}} \\ &= \frac{-3\sqrt{34}}{34} \end{aligned}$$

$$\begin{aligned} \tan \theta &= \frac{5}{-3} \\ &= -\frac{5}{3} \end{aligned}$$

3. One of the primary trigonometric ratios for an angle is given, as well as the quadrant in which each angle is located. Find the other two trigonometric ratios of the angle.

a) $\sin A = \frac{3}{4}$, first quadrant

b) $\cos B = -\frac{2}{3}$, second quadrant

c) $\tan C = \frac{9}{11}$, third quadrant

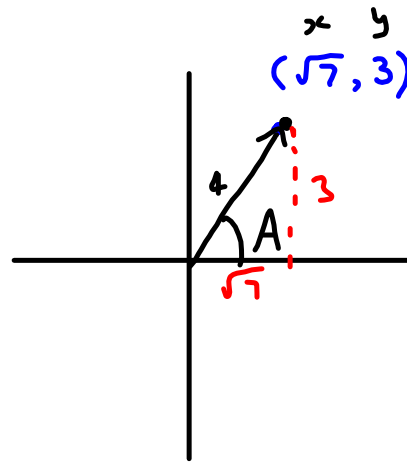
d) $\tan D = -\frac{5}{12}$, fourth quadrant

$$\begin{aligned} a) \quad x &= \sqrt{r^2 - y^2} \\ &= \sqrt{4^2 - 3^2} \\ &= \sqrt{7} \end{aligned}$$

(12, -5)

$$\cos A = \frac{\sqrt{7}}{4}$$

$$\begin{aligned} \tan A &= \frac{3}{\sqrt{7}} \\ &= \frac{3\sqrt{7}}{7} \end{aligned}$$

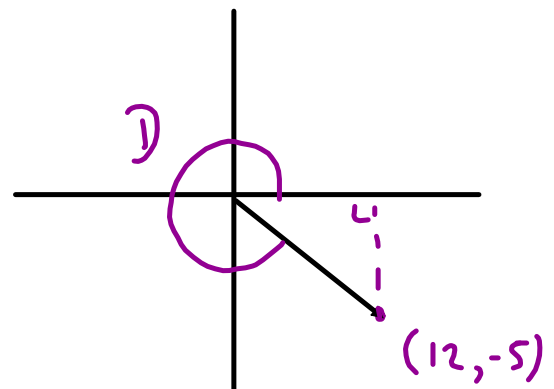


$$\begin{aligned} d) \quad r &= \sqrt{12^2 + (-5)^2} \\ &= \underline{\underline{13}} \end{aligned}$$

$$\tan D = -\frac{5}{12}$$

$$\sin D = -\frac{5}{13}$$

$$\cos D = \frac{12}{13}$$



4. Determine two other angles that have the same trigonometric ratios as each given angle.

a) $\sin 60^\circ$

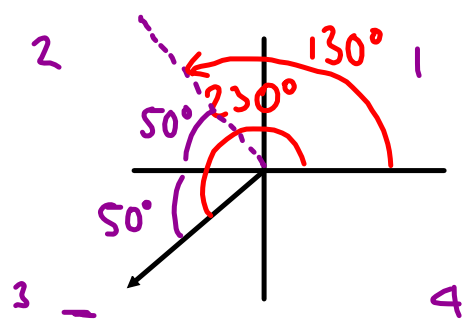
b) $\cos 230^\circ$

c) $\tan 200^\circ$

d) $\sin 150^\circ$

$$\begin{aligned} \sin 60^\circ &= \sin 120^\circ \\ &= \sin(-300^\circ) \end{aligned}$$

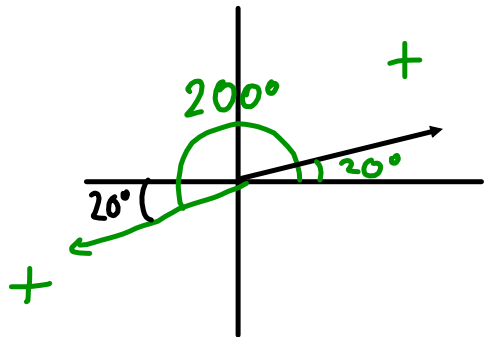
$\therefore 120^\circ$ and -300° have the same sine ratio (multiple answers)



$$\cos 230^\circ = \cos 130^\circ$$

$$\begin{aligned} &= \cos -490^\circ \\ &\text{(answers vary)} \end{aligned}$$

$\therefore 130^\circ$ and -490° have the same cosine ratio.



$$\begin{aligned} \tan 200^\circ &= \tan(-160^\circ) \\ &= \tan 20^\circ \end{aligned}$$

$\therefore -160^\circ$ and 20° have the same tangent ratio.

5. Determine any three positive angles that are co-terminal with 150° .

$$150^\circ, 510^\circ, 870^\circ, 1230^\circ$$

$(150+360) \quad (510+360)$

6. An obtuse angle θ has the point $B(-x, y)$ on its terminal arm.

a) Express the length of line segment OB in terms of x and y .

$$OB = \sqrt{(-x)^2 + y^2}$$

b) Write expressions for the primary trigonometric ratios for the angle.

$$\sin \theta = \frac{y}{OB}$$

$$\cos \theta = \frac{-x}{OB}$$

$$\tan \theta = \frac{y}{-x}$$

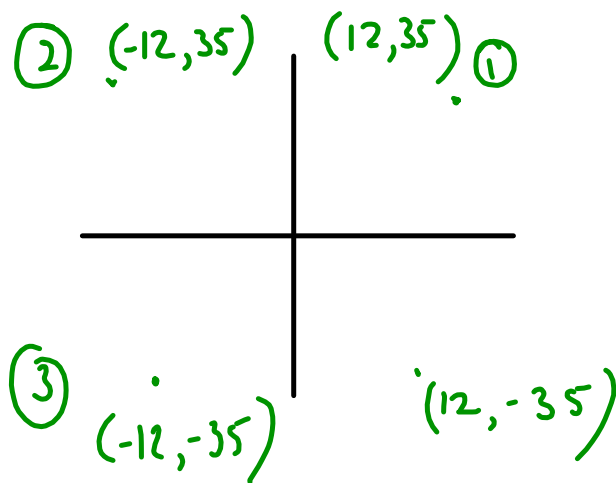
7. Consider $\angle F$ such that $\cos F = \frac{12}{37}$. \oplus

a) Which quadrants can $\angle F$ be in? 1st and 4th

b) Find the coordinates of a point on the terminal arm of the angle in each quadrant.

c) If you are also told that the sine of the angle is negative, in which quadrant is $\angle F$? 3rd or 4th? but 4th because cose is positive.

d) Write the other primary trigonometric ratios for $\angle F$ in the quadrant identified in part c.



$$\begin{aligned}
 y &= \sqrt{37^2 - 12^2} \\
 &= \sqrt{1225} \\
 &= \underline{35}
 \end{aligned}$$

Thursday: Quiz (5.1-5.4)
 Friday: Trig Identities task
Homework: Page 299 # 1-8, 12

