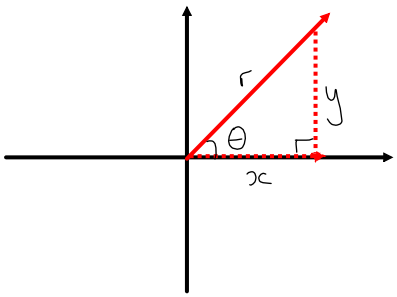
5.5 Proving Trigonometric Identities

Basic Identities can be verified using a standard right triangle in the first quadrant on a circle:



$\cos\theta = \frac{x}{r}$

$\therefore x = r\cos\theta$

$\sin\theta = \frac{y}{r}$

$\therefore y = r\sin\theta$

$\tan\theta = \frac{y}{x}$

$= \frac{r\sin\theta}{r\cos\theta}$

$$\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$$ ← Quotient Identity

$(\cos\theta)^2 = \cos^2\theta$

$x^2 + y^2 = r^2$

$(r\cos\theta)^2 + (r\sin\theta)^2 = r^2$

$\frac{r^2\cos^2\theta}{r^2} + \frac{r^2\sin^2\theta}{r^2} = \frac{r^2}{r^2}$

$\div r^2$

$$\boxed{\cos^2\theta + \sin^2\theta = 1}$$ ← Pythagorean Identity

These basic identities (quotient and pythagorean) are used to prove other trigonometric identities. We also use our reciprocal identities in our proofs:

$\cot\theta = \frac{\cos\theta}{\sin\theta}$, $\sec\theta = \frac{1}{\cos\theta}$, $\csc\theta = \frac{1}{\sin\theta}$

Prove $\frac{\cos x \tan x}{\sin x} = 1$

when proving trig identities, we must separate Left Side and Right Side.

LS

$\frac{\cos x \tan x}{\sin x}$

$= \cos x \cdot \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$

$= \frac{\sin x \cos x}{\sin x \cos x}$

$= 1$

RS

1

$\therefore$ LS = RS

Q.E.D.

(2) RTP $\frac{1}{\cos x} - \sin x \tan x = \cos x$

required to prove

LS

$\frac{1}{\cos x} - \sin x \tan x$

$= \frac{1}{\cos x} - \sin x \frac{\sin x}{\cos x}$

$= \frac{1}{\cos x} - \frac{\sin^2 x}{\cos x}$

$= \frac{1 - \sin^2 x}{\cos x}$

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$= \frac{\cos^2 x}{\cos x}$

$= \cos x$

RS

$\cos x$

$\therefore$ LS = RS

Q.E.D.

③ Prove $\frac{1}{\cot^2 x} - \frac{1}{\sec^2 x} = \sec^2 x - \cos^2 x - 1$

LS

$$\frac{1}{\cot^2 x} - \frac{1}{\sec^2 x}$$

$$= \tan^2 x - \cos^2 x$$

$$= \frac{\sin^2 x}{\cos^2 x} - \cos^2 x$$

$$= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^4 x}{\cos^2 x}$$

$$= \frac{\sin^2 x - \cos^4 x}{\cos^2 x}$$

$$= \frac{-\cos^4 x + (1 - \cos^2 x)}{\cos^2 x}$$

$$= \frac{-\cos^4 x - \cos^2 x + 1}{\cos^2 x}$$

RS

$$\sec^2 x - \cos^2 x - 1$$

$$= \frac{1}{\cos^2 x} - \cos^2 x - 1$$

$$= \frac{1}{\cos^2 x} - \frac{\cos^4 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$$

$$= \frac{-\cos^4 x - \cos^2 x + 1}{\cos^2 x}$$

$\therefore$ LS = RS

Q.E.D.

Homework

Prove the following identities

1. $\sec A + \tan A = \dfrac{1+\sin A}{\cos A}$
2. $\tan A + \cot A = \sec A \operatorname{cosec} A$
3. $\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2\theta \operatorname{cosec}^2\theta$
4. $\dfrac{\operatorname{cosec}\theta - \cot\theta}{1-\cos\theta} = \operatorname{cosec}\theta$
5. $\operatorname{cosec} x - \sin x = \cos x \cot x$
6. $1 + \cos^4 x - \sin^4 x = 2\cos^2 x$
7. $\sec\theta + \tan\theta = \dfrac{\cos\theta}{1-\sin\theta}$
8. $\dfrac{\sin A \tan A}{1-\cos A} = 1 + \sec A$