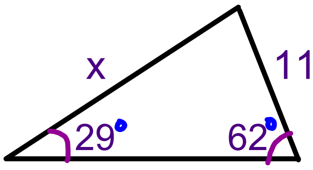
5.6 The Sine Law and The Ambiguous Case

Warm Up - Use the Sine Law to find the side length:

a)

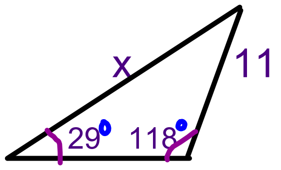


$$\frac{x}{\sin 62^\circ} = \frac{11}{\sin 29^\circ}$$

$$x = \frac{11\sin 62^\circ}{\sin 29^\circ}$$

$$x \doteq 20.0$$

b)



$$\frac{x}{\sin 118^\circ} = \frac{11}{\sin 29^\circ}$$

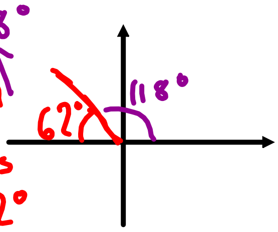
$$x = \frac{11\sin 118^\circ}{\sin 29^\circ}$$

$$x \doteq 20.0$$

x is the same value in both examples because

$\sin 62^\circ = \sin 118^\circ$

Related acute angle is 62°



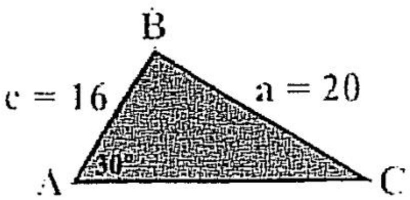
https://www.youtube.com/watch?feature=player_embedded&v=Io3xZMOrbkQ

Facts we need to remember:

1. In a triangle, the sum of the interior angles is 180°.
2. No triangles can have two obtuse angles.
3. The sine function has a range of $-1 \le \sin\theta \le 1$.
4. If the $\sin\theta$ = positive decimal < 1, the θ can lie in the first quadrant (acute <) or in the second quadrant (obtuse <).

Let's look at some cases. In each example, decide whether the given information points to the existence of one triangle, two triangles or no triangles.

Example 1: In $\triangle ABC$, $a = 20$, $c = 16$, and $m\angle A = 30°$. How many distinct triangles can be drawn given these measurements?



Use the Law of Sines:

$$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{20}{\sin 30°} = \frac{16}{\sin C}$$

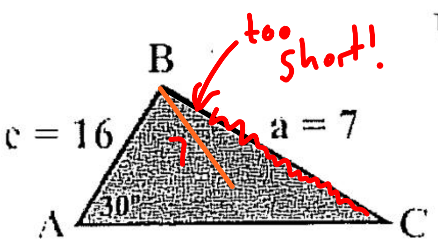
$$20(\sin C) = 16 \cdot \sin 30°$$

$$\sin C = \frac{16 \cdot (0.5)}{20} = 0.4$$

$C = \sin^{-1}(0.4) = 24°$ (to the *nearest degree*) - in Quadrant I.
Sine is also positive in Quadrant II. If we use the reference angle 24° in Quadrant II, the angle C is 156°.
related acute angle
But, with $m\angle A = 30°$ and $m\angle C = 156°$ the sum of the angles would exceed 180°.
Not possible!!!!

Therefore, $m\angle C = 24°$, $m\angle A = 30°$, and $m\angle B = 126°$ and only ONE triangle is possible.

Example 2: In $\triangle ABC$, $a = 7$, $c = 16$, and $m\angle A = 30°$. How many distinct triangles can be drawn given these measurements?



Use the Law of Sines:

$$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{7}{\sin 30°} = \frac{16}{\sin C}$$

$$7(\sin C) = 16 \cdot \sin 30°$$

$$\sin C = \frac{16 \cdot (0.5)}{7} = 1.1428$$

$C = \sin^{-1}(1.1428)$

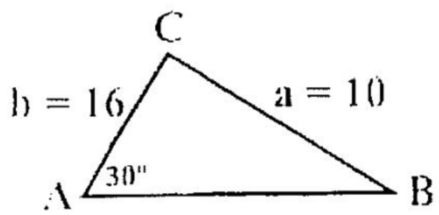
Since $\sin C$ must be ≤ 1, no angle exists for angle C.
NO triangle exists for these measurements.

error!

Example 3: In $\triangle ABC$, $a = 10$, $b = 16$, and $m\angle A = 30^\circ$. How many distinct triangles can be drawn given these measurements?

Use the Law of Sines:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{10}{\sin 30^\circ} = \frac{16}{\sin B}$$



$$10(\sin B) = 16 \cdot \sin 30^\circ$$

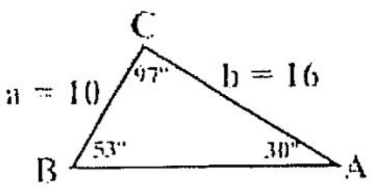
$$\sin B = \frac{16 \cdot (0.5)}{10} = 0.8$$

$B = \sin^{-1}(.8) = 53.13010 = 53^\circ$.

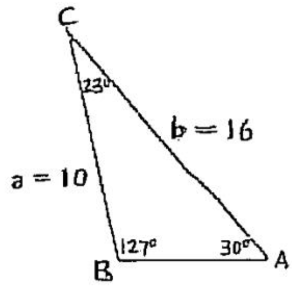
Angles could be 30°, 53°, and 97° : sum 180°

The angle from Quadrant II could create angles 30°, 127°, and 23° : sum 180°

180°-53°



TWO triangles possible.



This example is the **Ambiguous Case.** The information given is the postulate SSA (or ASS, the Donkey Theorem), but the two triangles that were created are clearly not congruent. We have two triangles with two sides and the non-included angle congruent, but the triangles are not congruent to each other.

Homework p318 #3,4,5,7

p339 #8-11