

5.7 Cosine Law

p325 # 1(a), 2(a), 3(b), 4 - 6, 10

In Summary

Key Idea

- Given any triangle, the cosine law can be used if you know
 - two sides and the angle contained between those sides (SAS) or
 - all three sides (SSS)

Need to Know

- The cosine law states that in any $\triangle ABC$,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

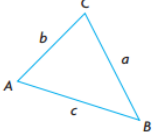
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$
- If $\angle A = 90^\circ$ and $\angle A$ is the contained angle, then the cosine law simplifies to the Pythagorean theorem:

$$a^2 = b^2 + c^2 - 2bc \cos 90^\circ$$

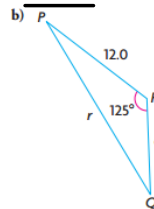
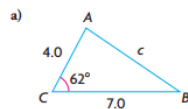
$$a^2 = b^2 + c^2 - 2bc(0)$$

$$a^2 = b^2 + c^2$$



CHECK Your Understanding

1. Determine each unknown side length to the nearest tenth.



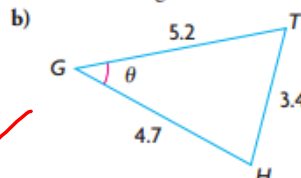
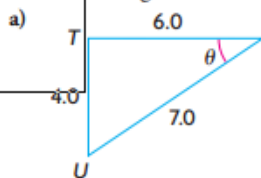
$$r^2 = 9^2 + 12^2 - 2(9)(12) \cos 125^\circ$$

$$= 81 + 144 - 216 \cos 125^\circ$$

$$r = \sqrt{225 - 216 \cos 125^\circ}$$

$$= \underline{18.7 \text{ units}}$$

2. For each triangle, determine the value of θ to the nearest degree.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$2bc \cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

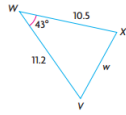
$$\cos G = \frac{5.2^2 + 4.7^2 - 3.4^2}{2(5.2)(4.7)}$$

$$= 0.768617...$$

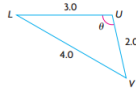
$$G = \cos^{-1}(0.768617)$$

$$\approx \underline{\underline{40^\circ}}$$

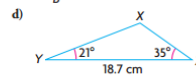
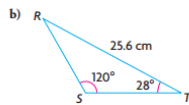
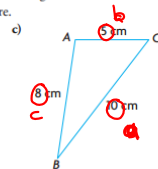
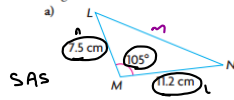
3. a) Determine w to the nearest tenth.



b) Determine the value of θ to the nearest degree.



4. Calculate each unknown angle to the nearest degree and each unknown length to the nearest tenth of a centimetre.



$$a) \quad a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$m^2 = n^2 + l^2 - 2nl \cdot \cos M$$

$$m^2 = 7.5^2 + 11.2^2 - 2(7.5)(11.2)\cos 105^\circ$$

$$= 56.25 + 125.44 - (-43.4816 \dots)$$

$$m^2 = 225.17159 \dots$$

$$m = \sqrt{225.17159 \dots}$$

$$m \approx 15.0 \text{ cm}$$

$$\frac{\sin N}{n} = \frac{\sin M}{m}$$

$$\frac{\sin N}{7.5} = \frac{\sin 105^\circ}{15.0}$$

$$\sin N = \frac{7.5 \sin 105^\circ}{15.0}$$

$$\approx 0.48296 \dots$$

$$N \approx \sin^{-1}(0.48296 \dots)$$

$$\therefore N \approx 29^\circ$$

$$L \approx 180^\circ - 105^\circ - 29^\circ$$

$$\therefore L \approx 46^\circ$$

b)

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$2bc \cdot \cos A = b^2 + c^2 - a^2$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{5^2 + 8^2 - 10^2}{2(5)(8)}$$

$$= \frac{25 + 64 - 100}{80}$$

$$= -\frac{11}{80}$$

$$A = \cos^{-1}\left(-\frac{11}{80}\right)$$

$$A \approx 98^\circ$$

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{5} = \frac{\sin 98^\circ}{10}$$

$$\sin B = \frac{5 \sin 98^\circ}{10}$$

$$\approx 0.49525 \dots$$

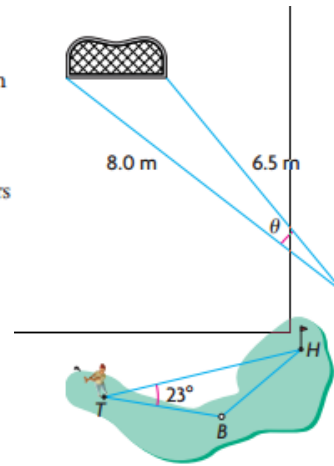
$$B \approx \sin^{-1}(0.49525 \dots)$$

$$\therefore B \approx 30^\circ$$

$$C \approx 180 - 30 - 98$$

$$\therefore C \approx 52^\circ$$

5. The posts of a hockey goal are 2.0 m apart. A player attempts to score by shooting the puck along the ice from a point 6.5 m from one post and 8.0 m from the other. Within what angle θ must the shot be made? Round your answer to the nearest degree.
- A**
6. While golfing, Sahar hits a tee shot from T toward a hole at H , but the ball veers 23° and lands at B . The scorecard says that H is 270 m from T . If Sahar walks 160 m to the ball (B), how far, to the nearest metre, is the ball from the hole?



10. The Leaning Tower of Pisa is 55.9 m tall and leans 5.5° from the vertical. If its shadow is 90.0 m long, what is the distance from the top of the tower to the top edge of its shadow? Assume that the ground around the tower is level. Round your answer to the nearest metre.