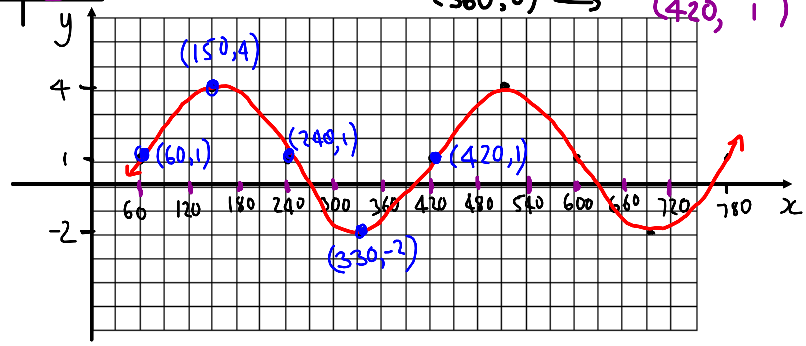
6.5 Using transformations to sketch the Graphs of Sinusoidal Functions

1. Create a graph of the function $y = 3\sin(x - 60°) + 1$ for two cycles. (Method 1)

a, d, c

	x	y
R		
S		3
T	60	1

$y = \sin x$		$y = 3\sin(x - 60°) + 1$
(0, 0)	$\longrightarrow$	(60, 1)
(90, 1)	$\longrightarrow$	(150, 4)
(180, 0)	$\longrightarrow$	(240, 1)
(270, −1)	$\longrightarrow$	(330, −2)
(360, 0)	$\longrightarrow$	(420, 1)



$$\text{Period} = \frac{360°}{k} = \frac{360°}{1} = \underline{360°}$$

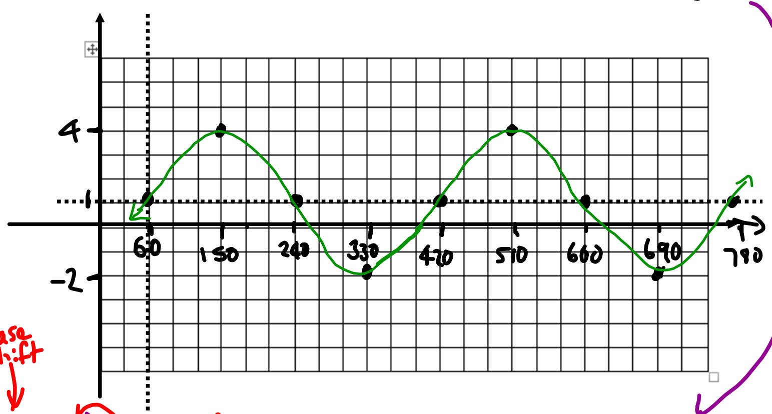
1. Create a graph of the function $y = 3\sin(x - 60°) + 1$ for two cycles. (Method 2)

amplitude = 3

equation of the axis is $y = 1$

phase shift = 60°

$$\text{Period} = \frac{360°}{k} = \frac{360°}{1} = 360°$$



phase shift

(60, 1) eqn of axis value

+90 +3

(150, 4)

+90

(240, 1)

+90 −3

(330, −2)

+90°

(420, 1)

Key points are in increments of 90° $\left(\frac{360°}{4}\right)$

2. Create a graph of the function $y = \frac{1}{2}\cos\left[\frac{1}{2}(x - 30°)\right] - 2$ for two cycles

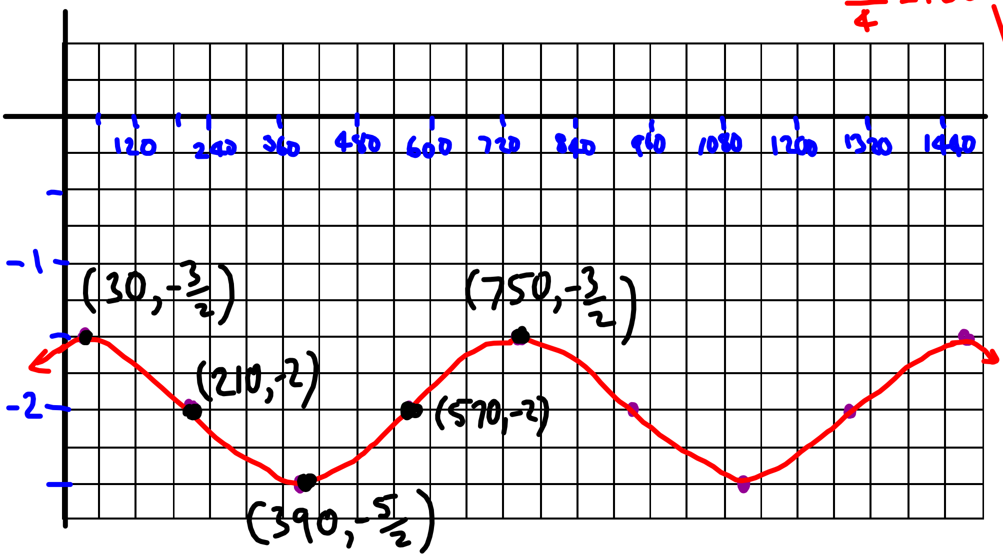
amplitude $= \frac{1}{2}$

eqn of axis $= -2$

phase shift $= 30$

period $= \frac{360}{K}$ $= 360 \div \frac{1}{2}$ $= \underline{720}$

$\frac{720}{4} = 180$



Key Points

$(30, -\frac{3}{2})$

+180

$(210, -2)$

$(390, -\frac{5}{2})$

$(570, -2$

$(750, -\frac{3}{2})$

	x	y
R		
S	2	$\frac{1}{2}$
T	30	-2

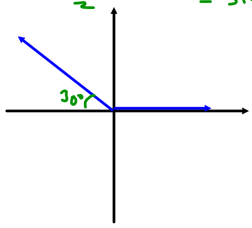
$y = \cos x$		$y = \frac{1}{2}\cos\left[\frac{1}{2}(x - 30°)\right] - 2$
$(0, 1)$	$\rightarrow$	$(30, -\frac{3}{2})$
$(90, 0)$	$\rightarrow$	$(210, -2)$
$(180, -1)$	$\rightarrow$	$(390, -\frac{5}{2})$
$(270, 0)$	$\rightarrow$	$(570, -2)$
$(360, 1)$	$\rightarrow$	$(750, -\frac{3}{2})$

UNIT TEST (takeup)

b) sin(-210°) $= \sin(-210+360)$

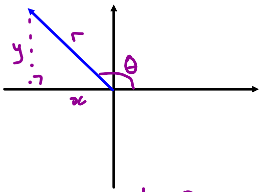
$= \sin(150°)$

$= \frac{1}{2}$



3. Given **θ** is a principal angle in the second quadrant, and sin**θ** = $\frac{2}{3}$ $\frac{y}{r}$, write the exact values of cos**θ** and tan**θ**. ***3 marks***

@2



$x = \sqrt{r^2 - y^2}$

$= \sqrt{3^2 - 2^2}$

$= -\sqrt{5}$

$\cos\theta = \frac{-\sqrt{5}}{3}$

$= -\frac{\sqrt{5}}{3}$

$\tan\theta = \frac{2}{-\sqrt{5}}$

$= -\frac{2\sqrt{5}}{5}$

$\cos 60^\circ + \csc 30^\circ + 2\sin 45^\circ \cos 45^\circ$

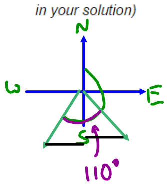
$= \frac{1}{2} + (1 \div \frac{1}{2}) + 2\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

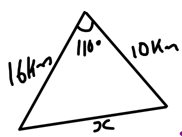
$= \frac{1}{2} + 2 + \frac{4}{4}$

$= \frac{1}{2} + \frac{4}{2} + \frac{2}{2}$

Application

5. A rescue boat travelling at 8 km/h departs from Port Stanley towards an island at a bearing of 235°. At the same time, a cruise ship travelling at 5 km/h heads out to sea from Port Stanley at a bearing of 125°. After 2 hours how far apart (to the nearest tenth of a kilometre) are the rescue boat and cruise ship? *(include a labelled diagram in your solution)* ***4 marks***

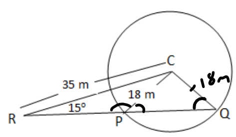




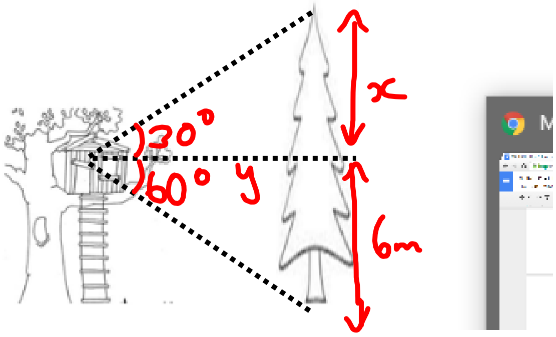
$x = \sqrt{10^2 + 16^2 - 2(10)(16)\cos 110^\circ}$

$\doteq 21.6\text{km}$

6. Point C in the diagram below represents the centre of a circle with radius 18 m. Calculate the length of line segment PQ to the nearest tenth of a metre. ***5 marks***



PQ = 31.1m



$$\tan 60° = \frac{6}{y}$$

$$\sqrt{3} = \frac{6}{y}$$

$$y = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = \underline{2\sqrt{3}}$$

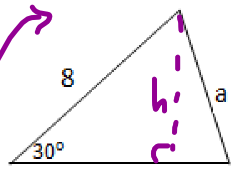
$$\tan 30° = \frac{x}{y}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{2\sqrt{3}}$$

$$\underline{x = 2}$$

Tree Height $= 6 + 2$
$= 8$ metres

1. Given the triangle shown below, explain what value(s) of **a** would allow:
a) 0 triangles, b) 1 triangle, and c) 2 triangles to be drawn. ***4 marks***



$$\sin 30° = \frac{h}{8}$$

$$h = 8 \sin 30° = \underline{4}$$

$a < 4$ ✓

$a = 4$ ✓
or $a \geq 8$ ✓

$4 < a < 8$ ✓

2. Explain the situations where the Sine Law must be used to solve a triangle. ***3 marks***

6.4 (takeup)

Factor	Value	Effect
a	$a > 1$	amplitude is greater than 1
	$0 < a < 1$	" " less than 1
	$-1 < a < 0$	" " " " "
	$a < -1$	" " greater than 1
k	$k > 1$	horizontal compression
	$0 < k < 1$	" stretched
	$-1 < k < 0$	" " , reflected through y-axis
	$k < -1$	" compression, " " "
d	$d > 0$	phase shift (left)
	$d < 0$	" " (right)
c	$c > 0$	vertical shift (up)
	$c < 0$	" " (down)

$$y = a\sin[k(x-d)] + c$$

$$y = a\cos[k(x-d)] + c$$

k is always factored out!