6.7 Applications of Sinusoidal Functions

1. During high tide, the water depth in a harbour is 20 m, and during low tide it is 8 m. Assume a 12 h cycle (ie. period).

↑ max ↑ min

a) State where the water level is at
   i. A maximum. 20 metres
   ii. A minimum 8 metres

b) Find the amplitude "a", the average sea level (ie. "c" the horizontal axis) and the "k" value. We will find the phase shift "d" later.

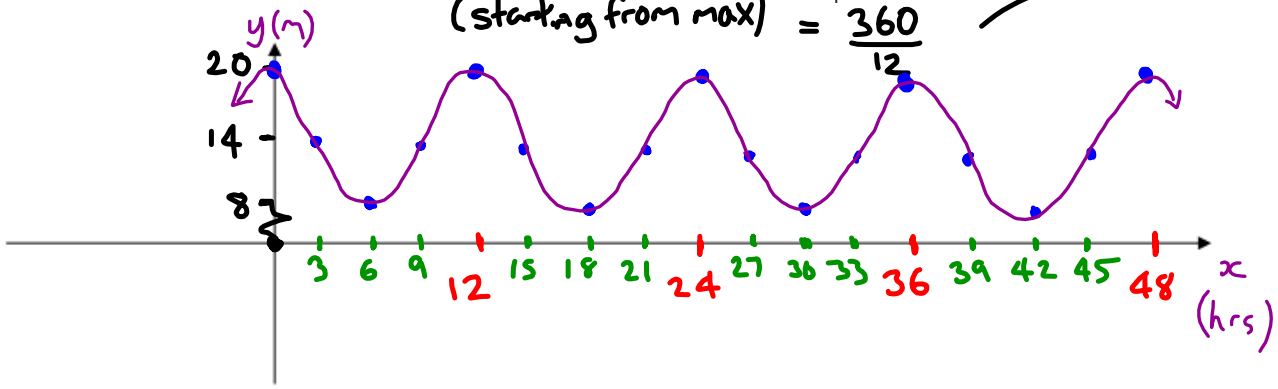
↑ aka The 'equation of the axis'

$a = \frac{\text{max}-\text{min}}{2} = \frac{20-8}{2} = 6 \text{ metres}$

$c = \frac{\text{max}+\text{min}}{2} = \frac{20+8}{2} = 14 \text{ metres}$

$K = \frac{360}{\text{period}}$ (starting from max) $= \frac{360}{12}$ → $K = 30$

c) Draw a graph of the function for a 48 h duration



d) From the graph, what is the phase shift "d"? Find an equation for water depth "y" metres for "x" hours

d=0 when modelling with 'cos'

$y = 6\cos[30x] + 14$

e) Calculate with your equation what the water depth will be at 57 hours.

let $x = 57$ →

$y = 6\cos((30)(57)) + 14$

$= 6\cos(1710) + 14$

$= 6(0) + 14$

$y = 14$

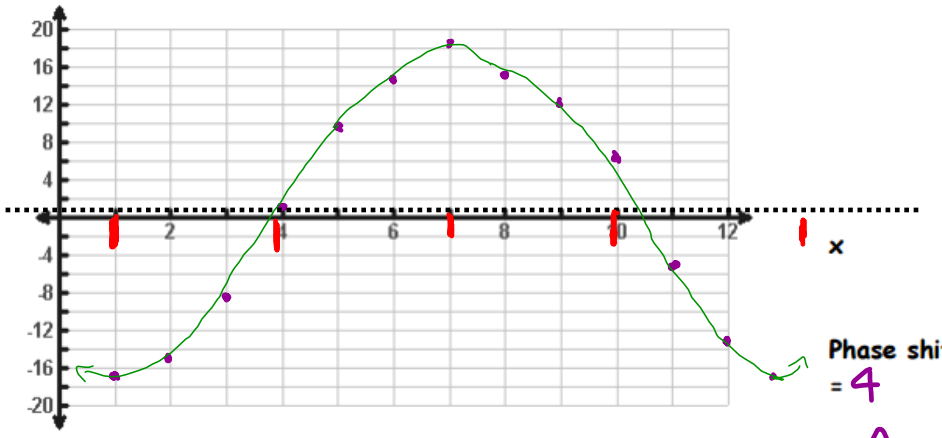
∴ The water depth at 57 hrs is 14 metres

2. The table shows the average monthly high temperature for one year that an environmentalist or weather forecaster would consider.

"x" time (months)	J 1	F	M	A	M	J	J	A	S	O	N	D
"y" temperature(°C)	-17.6	-15.2	-8.1	1.4	9.6	14.9	18.3	15.3	12.1	6.2	-5.3	-12.8

13
-17.6

(a) Draw a scatter plot of the data and the curve of best fit. January is month 1.



Phase shift "d"
= 4

(b) What type of model best describes the graph? Why?

either 'negative cos' (right shift 1 unit) or 'sine with a shift of 4 units right'

The graph hits the 'equation of the axis' and increases when $x = 4$

(c) Find the max, min, amplitude, average temperature "c", period, k value and phase shift "d" for this situation.

max = 18.3
min = -17.6

$a = \frac{18.3-(-17.6)}{2} = 17.95$

12 months

$k = \frac{360}{12} = 30$

$c = \frac{18.3+(-17.6)}{2} = 0.35$

(d) Find the equation of the sine model for the above situation using $y = a \sin k(x - d) + c$

$y = 17.95 \sin[30(x-4)] + 0.35$

(e) Use your equation to calculate the average monthly temperature for the 18th month, showing all your steps.

$x = 18 \rightarrow y = 17.95 \sin[30(18-4)] + 0.35$

$= 17.95 \sin(420) + 0.35$

$= 15.895°C$

p301 #15 Solving Trig. Equations

15. Given angle θ, where $0° \leq \theta \leq 360°$, solve for θ to the nearest degree.

a) $\cos 2\theta = 0.6420$

b) $\sin(\theta + 20°) = 0.2045$

c) $\tan(90° - 2\theta) = 1.6443$

a) $\cos 2\theta = 0.6420$

$2\theta = \cos^{-1}(0.6420)$

50+360 310+360

$2\theta = 50°, 310°, 410°, 670°$

Q1 Q4 Q1 Q4

$0° \leq \theta \leq 360°$

$0° \leq 2\theta \leq 720°$ $\therefore \theta = 25°, 155°, 205°, 335°$

b) $\sin(\theta + 20°) = 0.2045$

$0° \leq \theta \leq 360°$

$20° \leq \theta + 20° \leq 380°$

Homework: p398 #1, 3, 8, 9, 10, 11
p301 #15
complete handout 6.7