7.2 Geometric Sequences

A geometric sequence is a sequence where the ratio of consecutive terms is constant. (can be considered as a discrete exponential function)

Arithmetic Sequence

$t_2 - t_1 = d$

$t_3 - t_2 = d$

$t_4 - t_3 = d$

'd' is the common difference.
(linear)

Geometric Sequence

$\frac{t_2}{t_1} = r$

$\frac{t_3}{t_2} = r$

$\frac{t_4}{t_3} = r$

'r' is the common ratio
(exponential)

The general term for a Geometric Sequence is: $\boxed{t_n = ar^{n-1}}$

a: first term
r: common ratio
n: term number ($n \in \mathbb{N}$)

Examples

① Give $t_n = 3^{n-1}$, determine the first four terms.

$t_1 = 3^{1-1} = 3^0 = \underline{\underline{1}}$

$t_2 = 3^{2-1} = 3^1 = \underline{\underline{3}}$

$t_3 = 3^{3-1} = 3^2 = \underline{\underline{9}}$

$t_4 = 3^{4-1} = 3^3 = \underline{\underline{27}}$

$\therefore$ The first four terms are:

1, 3, 9, 27

The recursive formula for a Geometric Sequence is:

$$t_1 = a \quad , \boxed{t_n = r t_{n-1}} , \text{ where } n > 1$$

Ex. 2 Write the general term and recursive formula for the sequence, then state the value of t_5:

$$2, 6, 18, \ldots$$

$a = 2$

$r = \frac{6}{2}$

$r = 3$

$t_n = a r^{n-1}$

$$\boxed{\therefore t_n = 2(3^{n-1})}$$

$t_n = r t_{n-1}$

Recursively,

$$\boxed{t_1 = 2 \quad , \quad t_n = 3 t_{n-1} \quad , \; n > 1}$$

$t_5 = 2(3^{5-1})$

$= 2(3^4)$

$= 2(81)$

$\therefore t_5 = 162$

3 Find the number of terms in the sequence:

$$2, -6, 18, \ldots, 1458$$

$a = 2$

$r = \frac{18}{-6}$

$r = -3$

$t_n = 2(-3)^{n-1}$

$1458 = 2(-3)^{n-1}$

$729 = (-3)^{n-1}$

$(-3)^6 = (-3)^{n-1}$

$\therefore 6 = n - 1$

$n = 7$

$\therefore$ There are 7 terms in the sequence.

S
A
M ←
D
E ←
B

4 In a geometric sequence $t_3 = 99$ and $t_4 = 33$. Find the general term for t_n

$$t_n = a\left(\frac{1}{3}\right)^{n-1}$$

$$r = \frac{t_4}{t_3} = \frac{33}{99}$$

$$r = \frac{1}{3}$$

$$t_3 = 99 \rightarrow \quad 99 = a\left(\frac{1}{3}\right)^{3-1}$$

$$99 = a\left(\frac{1}{3}\right)^2$$

$$99 = a\left(\frac{1}{9}\right)$$

$$99 = \frac{a}{9}$$

$$a = 891$$

$$\therefore t_n = 891\left(\frac{1}{3}\right)^{n-1}$$

Homework: p430 #1-4, 6-10,12

H/W

$n \in \mathbb{N}$

9. i) Determine whether each general term defines an arithmetic sequence.
ii) If the sequence is arithmetic, state the first five terms and the common difference.

a) $t_n = 8 - 2n$ arithmetic

b) $t_n = n^2 - 3n + 7$ not arithmetic

c) $f(n) = \frac{1}{4}n + \frac{1}{2}$ arithmetic

d) $f(n) = \frac{2n + 5}{7 - 3n}$ not arithmetic

a) $t_1 = 8 - 2(1)$
$= 6$
$t_2 = 8 - 2(2)$
$= 4$
$t_3 = 8 - 2(3)$
$= 2$

$6, 4, 2, \ldots$

arithmetic
(common difference is -2)

d) $f(1) = \frac{2(1)+5}{7-3(1)}$
$= \frac{7}{4}$

$f(2) = \frac{2(2)+5}{7-3(2)}$
$= \frac{9}{1}$

$f(3) = \frac{2(3)+5}{7-3(3)}$
$= \frac{11}{-2}$

$f(4) = \frac{2(4)+5}{7-3(4)}$
$= \frac{13}{-5}$

$\frac{7}{4}, 9, -\frac{11}{2}, -\frac{13}{5}, \ldots$

not arithmetic