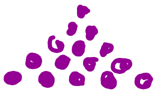
7.5 Arithmetic Series

Warm Up: Without using any of the formulas found in the textbook, calculate the sum of the consecutive integers from 1 to 100. (1 + 2 + ... + 100)

$$1 + 2 + \dots + 100$$



$$\begin{aligned} = &\ 100 + 99 + 98 + \dots + 51 \\ + &\ \ \ 1 + \ 2 + \ 3 + \dots + 50 \end{aligned}$$

$$= 50(101)$$

$$= \underline{5050}$$

A series is the sum of the terms of a sequence. For example, if an arithmetic sequence is 1, 3, 5, 7, ..., then the series would be 1 + 3 + 5 + 7 + ...

Deriving the Formula:

① $S_n = a + (a+d) + (a+2d) + \dots + (t_n - d) + t_n$

The sum of the first n terms

② $S_n = t_n + (t_n - d) + (t_n - 2d) + \dots + (a+d) + a$

① + ② → $2S_n = (a+t_n) + (a+t_n) + \dots + (a+t_n) + (a+t_n)$

$$2S_n = n(a+t_n)$$

$$S_n = \frac{n(a+t_n)}{2} \quad \text{or} \quad \boxed{S_n = \frac{n}{2}(a+t_n)}$$

but $t_n = a + (n-1)d$

$$S_n = \frac{n}{2}(a + a + (n-1)d)$$

$$\boxed{\therefore S_n = \frac{n}{2}(2a + (n-1)d)}$$

Ex1 Find the sum of the first 50 terms of the given series:

a) $3 + 8 + 13 + 18 + \dots$

b) $-1 - 5 - 9 - 13 - \dots$

a)

$a = 3$

$d = 8 - 3$

$d = 5$

$n = 50$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{50} = \frac{50}{2}(2(3) + (50-1)(5))$$

$= 25(6 + 49(5))$

$= 25(6 + 245)$

$= 25(251)$

$= 6275$

$\therefore$ The sum of the first 50 terms is 6275.

b)

$a = -1$

$d = -5 - (-1)$

$d = -4$

$n = 50$

$$S_{50} = \frac{50}{2}(2(-1) + (50-1)(-4))$$

$= 25(-2 + (49)(-4))$

$= 25(-2 + (-196))$

$= 25(-198)$

$= -4950$

$\therefore$ The sum of first 50 terms is -4950

Ex2 Find the sum of the series:

$3 + 7 + 11 + \dots + 479$

$t_n = 479$

$a = 3$

$d = 4$

$n?$

$t_n = a + (n-1)d$

$479 = 3 + (n-1)4$

$476 = (n-1)4$

$119 = (n-1)$

$120 = n$

S
A
M
D
E
B

$S_n = \frac{n}{2}(2a + (n-1)d) \rightarrow S_{120} = \frac{120}{2}(2(3) + (120-1)(4))$

$= 60(6 + (119)(4))$

$= 60(482)$

$= 28920$

$S_n = \frac{n}{2}(a + t_n)$

$\downarrow$

$S_{120} = \frac{120}{2}(3 + 479)$

$= 60(482)$

$= 28920$

$\therefore$ The sum of the series is 28920

Ex 3 Given the first and last terms, find the sum of each arithmetic series.

a) $t_1 = 3$, $t_{12} = 36$ $\quad S_n = \frac{n}{2}(a + t_n)$

$12 - 1 = 11$

$\therefore 11d = 36 - 3$

$11d = 33$

$\underline{d = 3}$

$a = 3$

$n = 12$

$S_{12} = \frac{12}{2}(3 + 36)$

$= 6(39)$

$= \underline{234}$

b) $t_1 = -4$, $t_{22} = -46$

ans: -550

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