

7.6 Geometric Series

A geometric series is the sum of the numbers in a Geometric Sequence.

eg. $1, 2, 4, 8, \dots$ is a geometric sequence.

$1 + 2 + 4 + 8 + \dots$ is a geometric series.

$$t_n = ar^{n-1}$$

Proving the Formula: $S_n = t_1 + t_2 + t_3 + \dots + t_n$

$$\textcircled{1} S_n = a + ar + ar^2 + \dots + ar^{n-1}$$

$$\textcircled{1} \times r \quad \textcircled{2} rS_n = ar + ar^2 + ar^3 + \dots + ar^n$$

$$\textcircled{1} - \textcircled{2} \rightarrow S_n - rS_n = a - ar^n$$

$$S_n(1 - r) = a(1 - r^n)$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

or
$$S_n = \frac{a(r^n - 1)}{r - 1}$$

You must know :

	sequence	series
arithmetic	$t_n = a + (n-1)d$	$S_n = \frac{n}{2}(2a + (n-1)d)$
	$t_n = t_{n-1} + d$	$S_n = \frac{n}{2}(a + t_n)$
geometric	$t_n = ar^{n-1}$	$S_n = \frac{a(r^n - 1)}{r - 1}$
	$t_n = rt_{n-1}$	$S_n = \frac{t_{n+1} - t_1}{r - 1}$

alternate formula (rarely used)

Ex1 Find the sum of the first 8 terms of the geometric series

$$1 + 4 + 16 + \dots$$

$$\begin{aligned} r &= 4 \\ a &= 1 \\ n &= 8 \end{aligned}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

ans: 21845

Ex2 Find the sum of the given series:
 $16 + 4 + 1 + \dots + 0.015625$

Homework
 P459
 #1, 3,
 5-9
 ALSO
 complete
 practice
 test

$$\begin{aligned} t_n &= ar^{n-1} & r &= \frac{1}{4} \\ 0.015625 &= 16\left(\frac{1}{4}\right)^{n-1} & a &= 16 \\ \frac{1}{64} &= 16\left(\frac{1}{4}\right)^{n-1} & n &=? \\ \frac{1}{(64)(16)} &= \left(\frac{1}{4}\right)^{n-1} \\ \frac{1}{1024} &= \left(\frac{1}{4}\right)^{n-1} \\ \left(\frac{1}{4}\right)^5 &= \left(\frac{1}{4}\right)^{n-1} \end{aligned}$$

$\left. \begin{array}{l} \rightarrow \\ \rightarrow \\ \rightarrow \end{array} \right\} \begin{array}{l} 5 = n - 1 \\ \underline{n = 6} \end{array}$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_6 = \frac{16\left(\left(\frac{1}{4}\right)^6 - 1\right)}{\left(\frac{1}{4} - 1\right)}$$

$$= \frac{16\left(\frac{1}{4096} - 1\right)}{-\frac{3}{4}}$$

$$= 16\left(\frac{-4095}{4096}\right) \times \frac{-4}{3}$$

$$= \frac{262080}{12288}$$

$$= \underline{\underline{21.328125}}$$