8.4 Solving Exponential Equations using Same Base
(not in textbook)

Example 1

Change the Base of a Power

Rewrite each power with the indicated base.

a) 9^2 as a power of 3 **b)** 2^4 as a power of 4

Solution

a) $9^2 = (3^2)^2$ — $3^2 = 9$, so substitute 3^2 for 9 in the expression 9^2.

$= 3^4$ — Apply the power of a power law.

Calculate to check that these expressions are equal.

$9^2 = 81 \qquad 3^4 = 81$

b) $2^4 = \left(4^{\frac{1}{2}}\right)^4$ — $2 = \sqrt{4}$ or $4^{\frac{1}{2}}$, so substitute $4^{\frac{1}{2}}$ for 2 into the expression 2^4.

$= 4^{\frac{4}{2}}$ — Apply the power of a power law.

$= 4^2$ — Simplify the exponent.

Example 2

Solve an Exponential Equation Involving Powers

Solve each exponential equation by writing powers with a common base.

a) $4^x = 64$

b) $3^{x+5} = 27^{x-1}$

c) $4^{2n-3} = 8^{n+1}$

Solution

a) $4^x = 64$ — Express the left side using base 4. 64 can be written as 4^3.

$4^x = 4^3$ — These equal powers have the same base, so the exponents are equal.

$x = 3$

b) $3^{x+5} = 27^{x-1}$

$3^{x+5} = (3^3)^{x-1}$ — 27 can be written as 3^3.

$3^{x+5} = 3^{3(x-1)}$ — Apply the power of a power law.

$3^{x+5} = 3^{3x-3}$ — Multiply the binomial by 3.

These powers are equal and have the same base. Their exponents must also be equal.

$x + 5 = 3x - 3$ — Set the exponents equal. Solve for x.

$8 = 2x$

$x = 4$

c) $4^{2n-3} = 8^{n+1}$ — Write 4 and 8 as powers of 2: $4 = 2^2$ and $8 = 2^3$.

$(2^2)^{2n-3} = (2^3)^{n+1}$ — Write both sides of the equation as powers of 2.

$2^{4n-6} = 2^{3n+3}$ — Apply the power of a power law.

$4n - 6 = 3n + 3$ — Set the exponents equal. Solve for n.

$4n - 3n = 3 + 6$

$n = 9$

Example 3 **Solve a Problem Involving Expressions With Powers**

Recall the equations for Raj's and Helen's scores from the Investigate:

Raj's score: $S = 2^d$ Helen's score: $S = 4^{(d-3)}$

Determine when Helen's score will equal Raj's score using an algebraic method.

Solution

To determine when Raj and Helen will have the same score, set their equations equal and solve for d, the number of days.

$2^d = 4^{(d-3)}$

$2^d = (2^2)^{(d-3)}$ Write the power on the right side in terms of base 2.

$2^d = 2^{2(d-3)}$ Apply the power of a power law.

$2^d = 2^{2d-6}$

$d = 2d - 6$

$d + 6 = 2d$

$6 = 2d - d$

$6 = d$

Raj and Helen will have the same score six days after Raj discovered the game.

Key Concepts

- Powers can be represented in various ways, using different base values.
- If two equal powers have the same base, then their exponents must also be equal.
- It is sometimes useful to change the base of an exponential expression when solving equations.

Discuss the Concepts

D1. There is more than one equivalent way to write a power. Is this statement true or false? Include two examples to support your answer.

D2. Which equation is equivalent to $2^{x+1} = 4^x$? How do you know?

A $2^{x+1} = 4^{2x}$ **B** $2^{x+1} = 2^{x+2}$

C $2^{x+1} = 2^{2x}$ **D** $2^{x+1} = (\sqrt{4})^x$

Practise A

For help with questions 1 to 3, refer to Example 1.

1. Write each power as a power with base 3.

a) 9^3 **b)** 27^2 **c)** 81^1 **d)** 6^0

2. Write each power as a power with base 10.

a) 100^4 b) $10\ 000^2$ c) 0.1

3. How are the values in parts a) and b) of question 2 related?

4. Write each power as a power with base 4.

a) 16^2 b) 64^3 c) 2^6 d) $(-16)^0$

For help with questions 5 to 7, refer to Example 2.

5. a) Solve $2^{x+1} = 4^x$.

b) Check your solution by substituting into the left and right sides of the equation and evaluating.

6. Solve.

a) $16^3 = 4^x$ b) $9^y = 3^4$
c) $3^{3x+1} = 9^{x-2}$ d) $25^{2y-1} = 5^{3y+1}$
e) $4^{2(p-1)} = 64^{3p+4}$ f) $1000^{2m-5} = 10^{3(m+3)}$

7. Solve. Check your solutions.

a) $4^{x-3} = 8^{x+1}$ b) $27^{w-3} = 9^{2(w+4)}$

10. Consider the equation $81^{3(x+1)} = 9^{2(x-1)}$.

a) Solve this equation by expressing both sides as powers of 3.

b) Solve this equation by expressing both sides as powers of 9.

c) How do your answers to parts a) and b) compare?

d) Which method do you prefer? Why?

11. Consider the equation $2^{x^2} = 4^{x^2-2}$.

a) There are two solutions to this equation.

Find the solutions algebraically. Check both solutions.

13. Consider the equation $2^{2x} = 6^x$.

a) Is it possible to solve this equation by writing both sides as powers that have the same base? If it is possible, solve the equation. If not, explain why.

b) If the equation is solvable, solve it using a different method. Explain your method and show your solution.

1. a) 3^6 b) 3^6 c) 3^4 d) 3^0
2. a) 10^8 b) 10^8 c) 10^{-1}
3. They are equal.
4. a) 4^4 b) 4^9 c) 4^3 d) 4^0
5. a) $x = 1$ b) LS = 4, RS = 4
6. a) $x = 6$ b) $y = 2$ c) $x = -5$
   d) $y = 3$ e) $p = -2$ f) $m = 8$
7. a) $x = -9$ b) $w = -25$
9. six days faster
10. a), b) $x = -2$
    c) They are equal. d) Answers may vary.
11. a), b) $x = -2, x = 2$

12. a) 100 insects b) 3200 insects c) 25 insects
    d) 10.3 days, 125 659 insects of each type
13. a) No; 2 and 6 do not have a common base.
    b) $x = 0$