Exam Review

The exam will be on every unit & every topic!

Make sure you review your unit tests, try the exam review handouts and self-tests from the textbook.

This is a lot of material to study, it will take time!

If $g(x) = 2x^2 + 2$, find a simplified expression for $g(2x + 1)$.

$$g(2x+1) = 2(2x+1)^2 + 2$$
$$= 2(4x^2+2x+2x+1) + 2$$
$$= 2(4x^2+4x+1) + 2$$
$$= 8x^2+8x+2+2$$
$$\therefore g(2x+1) = 8x^2 + 8x + 4$$

The graph of $y = \sqrt{x}$ is expanded vertically by a factor of 3, reflected in the x-axis, translated 2 units to the right, and translated 5 units downwards. What is the equation of the transformed function, g(x)?

a a c d

given f(x)

$$g(x) = af[k(x-d)] + c$$

- $\frac{1}{k}$ horizontal stretch/compression factor
- reflection through y-axis

vertical translation

- vertical stretch/compression factor
- reflection through x-axis

horizontal translation

<u>Answer</u>

Given $f(x)=\sqrt{x}$, $g(x) = -3f(x-2) - 5$

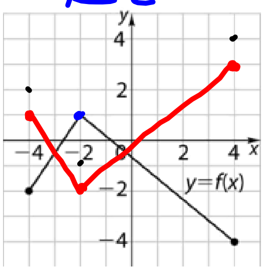
<u>or</u>

$$g(x) = -3\sqrt{x-2} - 5$$

The graph of $y = f(x)$ is shown. Sketch the following functions. Show your work.

b) $-f(x) - 1$

a c



KP $(f(x))$		KP $g(x)$
$(-4, -2)$	$\longrightarrow$	$(-4, 1)$
$(-2, 1)$	$\longrightarrow$	$(-2, -2)$
$(4, -4)$	$\longrightarrow$	$(4, 3)$

	X	Y
R		✓
S		-1
T		-1

1. Expand and simplify the following algebraic expressions:

b) $(4a^2b^2) \times (-2a^2b^3)$

$= (4)(-2)a^2a^2b^2b^3$

$= \underline{-8a^4b^5}$

2. Factor the following algebraic expressions:

a) $4x^2 + 16x + 16$ *2 marks*

$= 4(x^2+4x+4)$

$= 4(x+2)^2$

b) $6x^2 - x - 2$ *2 marks*

$= 6x^2-4x+3x-2$ $\quad -4 \times 3 = (6)(-2)$

$-4+3 = -1$

• common factor first!

$= 2x(3x-2)+1(3x-2)$

$= \underline{\underline{(3x-2)(2x+1)}}$

4. Simplify the following algebraic expression and state any restrictions:

c) $\dfrac{x^2-6x+5}{x^2+5x+4} \div \dfrac{x-1}{x^2+6x+8}$

$= \dfrac{(x-5)(x-1)}{(x+4)(x+1)} \div \dfrac{(x-1)}{(x+4)(x+2)}$

$= \dfrac{(x-5)\cancel{(x-1)}}{\cancel{(x+4)}(x+1)} \times \dfrac{\cancel{(x+4)}(x+2)}{\cancel{(x-1)}}$

$= \dfrac{(x-5)(x+2)}{x+1}$

restrictions

$x \neq -4, -2, -1, 1$

d) $\dfrac{4x}{2x-3} - \dfrac{7x}{3-2x}$

$= \dfrac{4x}{2x-3} - \dfrac{7x}{-(2x-3)}$

$= \dfrac{4x}{2x-3} + \dfrac{7x}{2x-3}$

$= \dfrac{4x+7x}{2x-3}$

$= \dfrac{11x}{2x-3}$; $2x-3 \neq 0$

$2x \neq 3$

$x \neq \frac{3}{2}$

$y = 3x^2 - 2x - 9$

$= 3[x^2 - \frac{2}{3}x] - 9$

$= 3[x^2 - \frac{2}{3}x + \frac{1}{9} - \frac{1}{9}] - 9$

$= 3[(x-\frac{1}{3})^2 - \frac{1}{9}] - 9$

$= 3(x-\frac{1}{3})^2 - \frac{3}{9} - 9$

$= 3(x-\frac{1}{3})^2 - \frac{28}{3}$

$y=0$

$0 = 3(x-\frac{1}{3})^2 - \frac{28}{3}$

$\frac{28}{3} = 3(x-\frac{1}{3})^2$

$\frac{28}{9} = (x-\frac{1}{3})^2$

$\pm\sqrt{\frac{28}{9}} = (x-\frac{1}{3})$

$x = \frac{1}{3} \pm \sqrt{\frac{28}{9}}$

Unit 3 - Quadratics

$y = ax^2 + bx + c$
standard form

a - direction of opening

c - y-intercept

$y = a(x-h)^2 + k$
vertex form

(h, k) vertex

$y = a(x-r)(x-s)$
factored form

r, s - 'zeroes' or 'roots' (x-intercepts)

Solving $0 = ax^2 + bx + c$

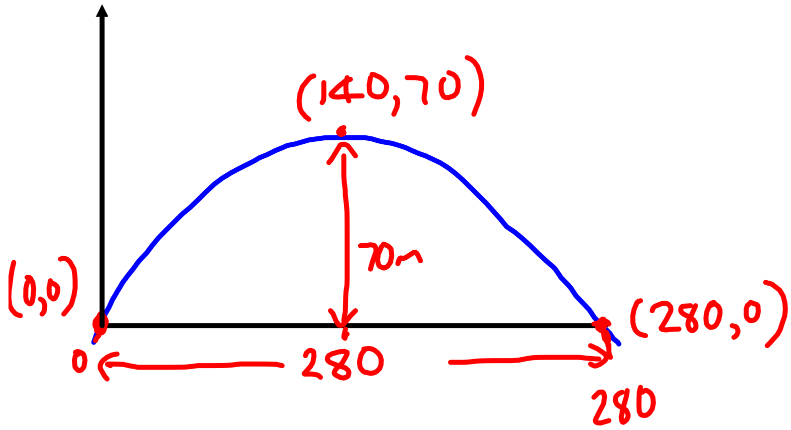
$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

$b^2 - 4ac > 0$ (2 roots)

$b^2 - 4ac = 0$ (1 root)

$b^2 - 4ac < 0$ (no roots)

2. A bridge is supported by an arch in the shape of a parabolic curve. The ends of the arch are on either side of the river that the bridge spans, 280 m apart. The top of the arch is at the centre of the bridge and is 70 m above the river. Determine the quadratic function that models the arch, **in factored or vertex form**, using one end of the arch as the origin (at (0, 0)). A diagram is required. ***(4 marks)***



$y = a(x-r)(x-s)$

$y = a(x-0)(x-280)$

$y = ax(x-280)$

sub $(140,70)$

$70 = a(140)(140-280)$

$70 = -19600a$

$-\frac{1}{240} = a$

$\therefore y = -\frac{1}{240}x(x-280)$

$y = a(x-h)^2 + k$

$y = a(x-140)^2 + 70$

sub $(0,0)$

$0 = a(0-140)^2 + 70$

$0 = 19600a + 70$

$-\frac{1}{240} = a$

$\therefore y = -\frac{1}{240}(x-140)^2 + 70$

Find the optimum value of the function, f(x) = -2x²+16x+178, state whether it is a maximum or minimum.

Complete the square
(standard → vertex form)

$$f(x) = -2x^2 + 16x + 178$$

$$= -2[x^2 - 8x] + 178$$

$$\left(\frac{-8}{2}\right)^2$$

$$= -2[x^2 - 8x + 16 - 16] + 178$$

$$= -2[(x-4)^2 - 16] + 178$$

$$= -2(x-4)^2 + 32 + 178$$

$$f(x) = -2(x-4)^2 + 210$$

∴ The optimum value is a maximum of 210

A canon ball is launched from a castle, the height of the ball (metres) over time (seconds) is modelled by the function, $f(x) = -0.2x^2+3.2x+21$.

a) ~~What is the domain and range~~ of this function?

b) When will the cannon ball be 30m high?

$$y = -0.2x^2 + 3.2x + 21$$

$$y = 30\text{m} \longrightarrow \quad 30 = -0.2x^2 + 3.2x + 21$$

$$0 = -0.2x^2 + 3.2x - 9$$

$$\times(-5) \rightarrow \quad 0 = x^2 - 16x + 45$$

$$0 = ax^2 + bx + c$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(45)}}{2(1)}$$

$$= \frac{16 \pm \sqrt{256 - 180}}{2}$$

$$= \frac{16 \pm \sqrt{76}}{2}$$

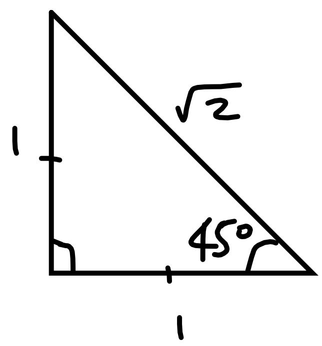
$$= \frac{16 \pm \sqrt{4}\sqrt{19}}{2}$$

$$= 8 \pm \sqrt{19}$$

$$\doteq 12.36, 3.64$$

∴ The cannonball reaches a height of 30m after approx 3.64 and 12.36 seconds

Unit 5 - Trig.



$\sin 45° = \frac{1}{\sqrt{2}}$ $\left(\text{or } \frac{\sqrt{2}}{2}\right)$

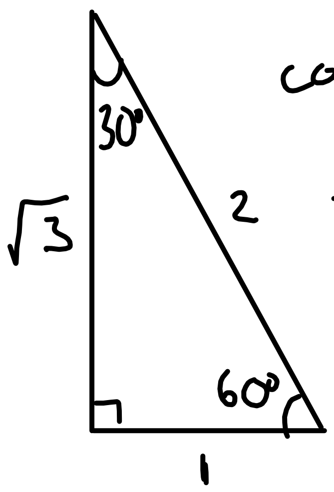
$\cos 45° = \frac{1}{\sqrt{2}}$ $\left(\text{or } \frac{\sqrt{2}}{2}\right)$

$\tan 45° = 1$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \sqrt{3}$



$\sin 30° = \frac{1}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$ $\left(\text{or } \frac{\sqrt{3}}{3}\right)$

1. State the exact values of the following: (5)

a. cos 120° $= -\cos 60°$

$= -\frac{1}{2}$

b. sin -330°

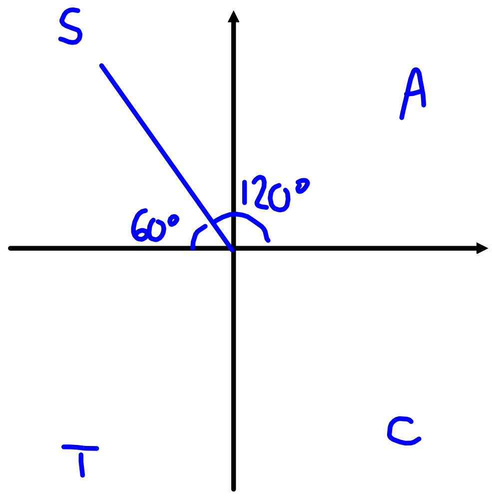
c. -tan 225°

d. csc 30° $= \frac{1}{\sin 30°}$

$= 1 \div \frac{1}{2}$

$= 2$

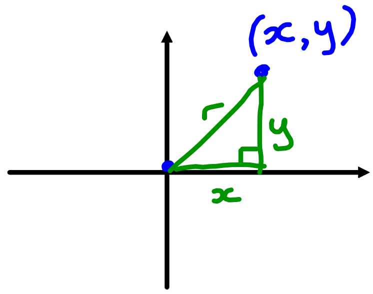
e. -sec -135°



$\sin\theta = \frac{y}{r}$ $\qquad \csc\theta = \frac{r}{y} = \frac{1}{\sin\theta}$

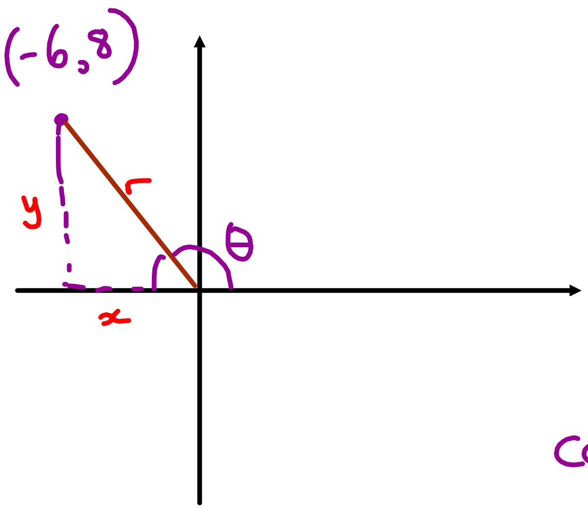
$\cos\theta = \frac{x}{r}$ $\qquad \sec\theta = \frac{r}{x} = \frac{1}{\cos\theta}$

$\tan\theta = \frac{y}{x}$ $\qquad \cot\theta = \frac{x}{y} = \frac{1}{\tan\theta}$



2. The point (-6, 8) is on the terminal arm of an angle ϴ in standard position.

Find sin ϴ and cos ϴ. *Leave as a simplified fraction with a rationalized denominator.* (3)



$\sin\theta = \frac{y}{r}$

$= \frac{8}{10}$

$= 0.8$

$r = \sqrt{x^2 + y^2}$

$= \sqrt{(-6)^2 + 8^2}$

$= \sqrt{36 + 64}$

$= 10$

$\cos\theta = \frac{x}{r}$

$= -\frac{6}{10}$

$= -0.6$

Unit 4 Exponential Functions

The value of a car after it is purchased depreciates according to the formula

$$V(n) = 28\,000(0.875)^n$$

where $V(n)$ is the car's value in the nth year since it was purchased.

a) What is the purchase price of the car? $\$28,000$
b) What is the annual rate of depreciation? 12.5%
c) What is the car's value at the end of 3 years?
d) What is its value at the end of 30 months?
e) How much value does the car lose in its first year?
f) How much value does it lose in its fifth year?

b) base $(1-r)$
(depreciation)

$1 - r = 0.875$
$r = 0.125$
$= 12.5\%$

c) $V(3) = 28000(0.875)^3$
$\doteq \$18757.81$

d) $n = \frac{30}{12}$
$= 2.5$

$V(2.5) = 28000(0.875)^{2.5}$
$\doteq \$20052.95$

$V(0) = 28000$
$V(1) = 28000(0.875)^1$
$= 24500$

loss in first year $= V(0) - V(1)$
$= 28000 - 24500$
$= \$3500$

e) $V(4) = 28000(0.875)^4$
$\doteq 16413.09$
$V(5) = 28000(0.875)^5$
$= 14361.45$

loss in fifth year $= V(4) - V(5)$
$= 16413.09$
$- 14361.45$
$\doteq \$2051.64$

Unit 6 Sinusoidal Functions

B
E
D
M
A
S

7. State the transformations, in the order you would have to apply them, to change the graph of $f(x) = \sin x$ into $g(x) = 4\sin[2(x+30)] + 16$ (2 marks)

$a=4$, $k=2$, $d=-30$, $c=16$

vertically stretched by a factor of 4.

$(\frac{1}{k}) \rightarrow$ horizontally compressed by a factor of $\frac{1}{2}$.

horizontally translation, 30 units left

vertical translation, 16 units up

7. Find the sum of the arithmetic series 6+13+20+...+132

- Use sequence formula to find n

t_n = the last value in a sequence or series

$t_n = 132$
$a = 6$
$d = 7$

$t_n = a + (n-1)d$
$132 = 6 + (n-1)(7)$
$132 = 6 + 7n - 7$
$132 = 7n - 1$
$133 = 7n$
$\underline{n = 19}$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{19} = \frac{19}{2}[2(6) + (19-1)(7)]$

$= 9.5[12 + (18)(7)]$

$= 9.5(138)$

$= \underline{\underline{1311}}$

∴ Sum of series is 1311

$y = a\,f[k(x - d)] + c$	
$t_n = a + (n-1)d$	$t_n = ar^{n-1}$
$S_n = \frac{n}{2}[2a + (n-1)d]$	$S_n = \frac{a(r^n - 1)}{r-1}$
$A = Prt + P$	$A = P(1+i)^n$
$FV = \frac{R[(1+i)^n - 1]}{i}$	$PV = \frac{R[1-(1+i)^{-n}]}{i}$

$P = A(1+i)^{-n}$

debt

4. You want to buy a \$1300 stereo on credit and make monthly payments over 2 years. If the store is charging you 18%/a compounded monthly, what will be your monthly payments?

$PV = \$1300$

$i = \frac{0.18}{12}$
$= 0.015$

$n = 12 \times 2$
$= \underline{24}$

$PV = \frac{R[1-(1+i)^{-n}]}{i}$

$\frac{PV\,i}{1-(1+i)^{-n}} = R$

$R = \frac{1300 \times 0.015}{1-(1+0.015)^{-24}}$

$\doteq \underline{\$64.90}$

5. Nazir saved \$900 to buy a plasma TV. He borrowed the rest at an interest rate of 18%/a compounded monthly. Two years later, he paid \$1429.50 for the principal and the interest. How much did the TV originally cost?

P ?

$A = 1429.50$

$n = 2 \times 12$

$= \underline{24}$

$i = \frac{0.18}{12}$

$= 0.015$

$P = A(1+i)^{-n}$

$= 1429.50(1+0.015)^{-24}$

$= 999.998$

$\doteq \$1000.00$

TV cost $= 900 + P$

$= 900 + 1000$

$= \underline{\$1900.00}$