

11U Functions Introduction to 1.8 – Combinations of Transformations

In mathematics, situations are rarely described by simple relationships, and so by combining translations, reflections, stretches, and compressions, you can model many different scenarios.

When combining transformations, order matters. To accurately sketch the graph of a function of the form $y = af[k(x - d)] + c$, apply transformations represented by the parameters a and k before transformations represented by the parameters d and c . That is, **stretches, compressions, and reflections occur before translations**. This is similar to the order of operations, where multiplication and division occur before addition and subtraction.

Stretches, compressions, and reflections can be performed in any order before translations.

- **Ensure that the function is written in the form $y = af[k(x - d)] + c$ to identify specific transformations.**
- **Transform the key points of the graph in question from the base graph to the transformed graph.**

Describe the roles of the parameters a , k , d , and c in the equation of a transformed function of the form $y = af[k(x - d)] + c$.

Summary of Transformations $y = af[k(x - d)] + c$		
Parameter	Description of Role	Graphical Example
a		
k		
d		
c		

Examples

1. For each function $g(x)$, and the base function $f(x)$, identify the values of a , k , d , and c .

a) $g(x) = 3f(x-1) - 2$ for $f(x) = x^2$

b) $g(x) = \frac{2}{3}f(2x-6) + 5$ for $f(x) = \sqrt{x}$

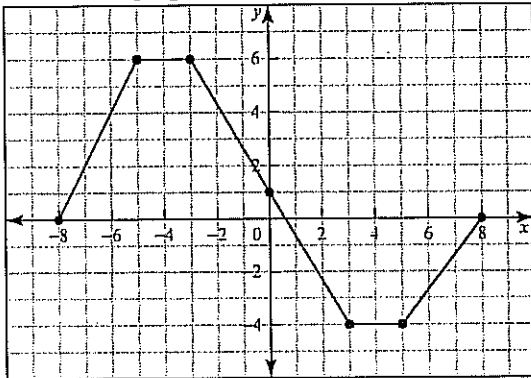
c) $g(x) = -2f(3x+9) - 2$ for $f(x) = x$

d) $g(x) = 4f(2x-1) + 1$ for $f(x) = \frac{1}{x}$

2. Identify the order in which the transformations must be applied to the base function to obtain the functions $g(x)$ in question 1.

3. Graph each base function and transformed function from question 1 on the same set of axes. Graph the transformed function using the order established in question 2.

4. For the graph of $f(x)$ given, sketch the graph of $g(x)$ after the given transformation.



a) $g(x) = 2f(x) - 2$

b) $g(x) = \frac{1}{2}f(x-1) + 1$

Practise

A

1. Describe how the graph of each of the following functions can be obtained from the graph of $y = f(x)$.

a) $y = 2f(x) + 3$ b) $y = \frac{1}{2}f(x) - 2$

c) $y = f(x + 4) + 1$ d) $y = 3f(x - 5)$

e) $y = f\left(\frac{1}{2}x\right) - 6$ f) $y = f(2x) + 1$

2. Describe how the graph of each of the following functions can be obtained from the graph of $y = f(x)$.

a) $y = -2f(x)$ b) $y = -\frac{1}{3}f(x)$

c) $y = f(-4x)$ d) $y = f\left(-\frac{1}{2}x\right)$

3. Describe how the graph of each of the following functions can be obtained from the graph of $y = f(x)$.

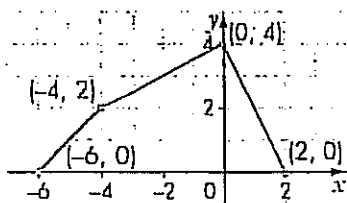
a) $y = -f(2x)$ b) $y = 3f(-2x)$

c) $y = -\frac{1}{2}f\left(\frac{1}{3}x\right)$ d) $y = 4f(x - 6) + 2$

e) $y = -2f(x) - 3$ f) $y = -f(x - 3) + 1$

g) $y = 3f(2x) - 6$ h) $y = \frac{1}{2}f\left(\frac{1}{2}x\right) - 4$

4. The graph of $y = f(x)$ is shown.



Sketch the graph of each of the following functions, state its domain and range, and identify any invariant points.

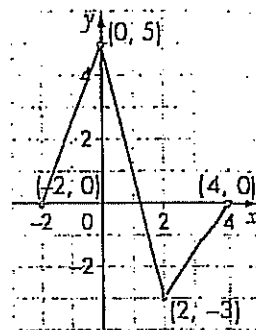
a) $y = f(x - 4) + 2$ b) $y = f(x + 2) - 4$

c) $y = \frac{1}{2}f(x) - 3$ d) $y = f(2x) + 3$

e) $y = -2f(x)$ f) $y = f(-x) - 2$

g) $y = f\left(-\frac{1}{2}x\right)$ h) $y = -\frac{1}{2}f(-2x)$

5. The graph of $y = f(x)$ is shown. Sketch the graph of each of the following functions, and state its domain and range.



a) $y = f(x - 3) + 3$

b) $y = -f(x) + 1$

c) $y = f(2x) - 3$

d) $y = 3f(x) - 2$

e) $y = f(-x) + 2$ f) $y = 2f(-x)$

6. Sketch each set of functions on the same set of axes in the given order.

a) $y = x$ b) $y = x^2$
 $y = 3x$ $y = 2x^2$
 $y = 3(x - 2) + 10$ $y = 2(x + 2)^2 - 3$

c) $y = x$ d) $y = x^2$
 $y = 0.5x$ $y = \left(\frac{1}{2}x^2\right)$
 $y = -0.5(x - 4) + 2$ $y = \frac{1}{2}(x + 3)^2 + 3$

e) $y = \sqrt{x}$ f) $y = \sqrt{x}$
 $y = \sqrt{2x}$ $y = 2\sqrt{x}$
 $y = -\sqrt{2x}$ $y = 2\sqrt{-x}$
 $y = -\sqrt{2(x - 1)} - 3$ $y = 2\sqrt{-(x - 3)} + 5$