Proving Trigonometric Identities

- Assignment Preparation (no notes!!!)

- Proper Form - seperate L.S. and R.S. and conclude with $\boxed{\therefore \text{ L.S.} = \text{R.S. Q.E.D.}}$

- Steps to proving:

  1. Rewrite all expressions in terms of $\sin\theta$ and $\cos\theta$

  2. When more than one term is given and one of the terms is rational, rewrite the expression as one simplified rational term (common denominator)

  3. Use 'factoring difference of squares' or 'multiplying by the conjugate' to simplify or rewrite rational expressions.

- Look for:

$$\boxed{\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ 1 - \sin^2\theta &= \cos^2\theta \\ 1 - \cos^2\theta &= \sin^2\theta \end{aligned}}$$

to simplify expressions

Multiplying by the "conjugate":

When given $(1-\sin\theta)$, $(1+\sin\theta)$, $(1-\cos\theta)$, $(1+\cos\theta)$ in a rational expression, multiply numerator and denominator by the conjugate of the factored binomial

eg. $(1+\cos\theta)$ is the conjugate of $(1-\cos\theta)$

(same 2 terms, but second term is opposite sign)

<u>Example</u> Prove $\dfrac{\sin\theta}{2(1-\cos\theta)} = \dfrac{1+\cos\theta}{2\sin\theta}$

L.S. $= \dfrac{\sin\theta}{2(1-\cos\theta)}$ R.S. $= \dfrac{1+\cos\theta}{2\sin\theta}$

$$= \frac{\sin\theta(1+\cos\theta)}{2(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{\sin\theta(1+\cos\theta)}{2(1-\cos^2\theta)}$$

$$= \frac{\cancel{\sin}\theta(1+\cos\theta)}{2\sin^{\cancel{2}}\theta}$$

$$= \frac{1+\cos\theta}{2\sin\theta}$$

$\therefore$ L.S = R.S

Q.E.D.

<u>Handout #4</u> $\dfrac{\csc\theta - \cot\theta}{1-\cos\theta} = \csc\theta$

L.S. $= \dfrac{\csc\theta - \cot\theta}{1-\cos\theta}$ R.S. $= \csc\theta$

$= \dfrac{1}{\sin\theta}$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right) \div (1-\cos\theta)$$

$$= \frac{1-\cos\theta}{\sin\theta} \times \frac{1}{1-\cos\theta}$$

$$= \frac{(1-\cancel{\cos\theta})}{\sin\theta(1-\cancel{\cos\theta})}$$

$$= \frac{1}{\sin\theta}$$

$\therefore$ L.S = R.S.

Handout #7

$$\sec\theta + \tan\theta = \frac{\cos\theta}{1-\sin\theta}$$

$$L.S. = \sec\theta + \tan\theta$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$$R.S. = \frac{\cos\theta}{1-\sin\theta}$$

$$= \frac{\cos\theta\,(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$$

$$= \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cancel{\cos\theta}(1+\sin\theta)}{\cos^{\cancel{2}}\theta}$$

$$= \frac{1+\sin\theta}{\cos\theta}$$

$\therefore L.S = R.S.$

Q.E.D.